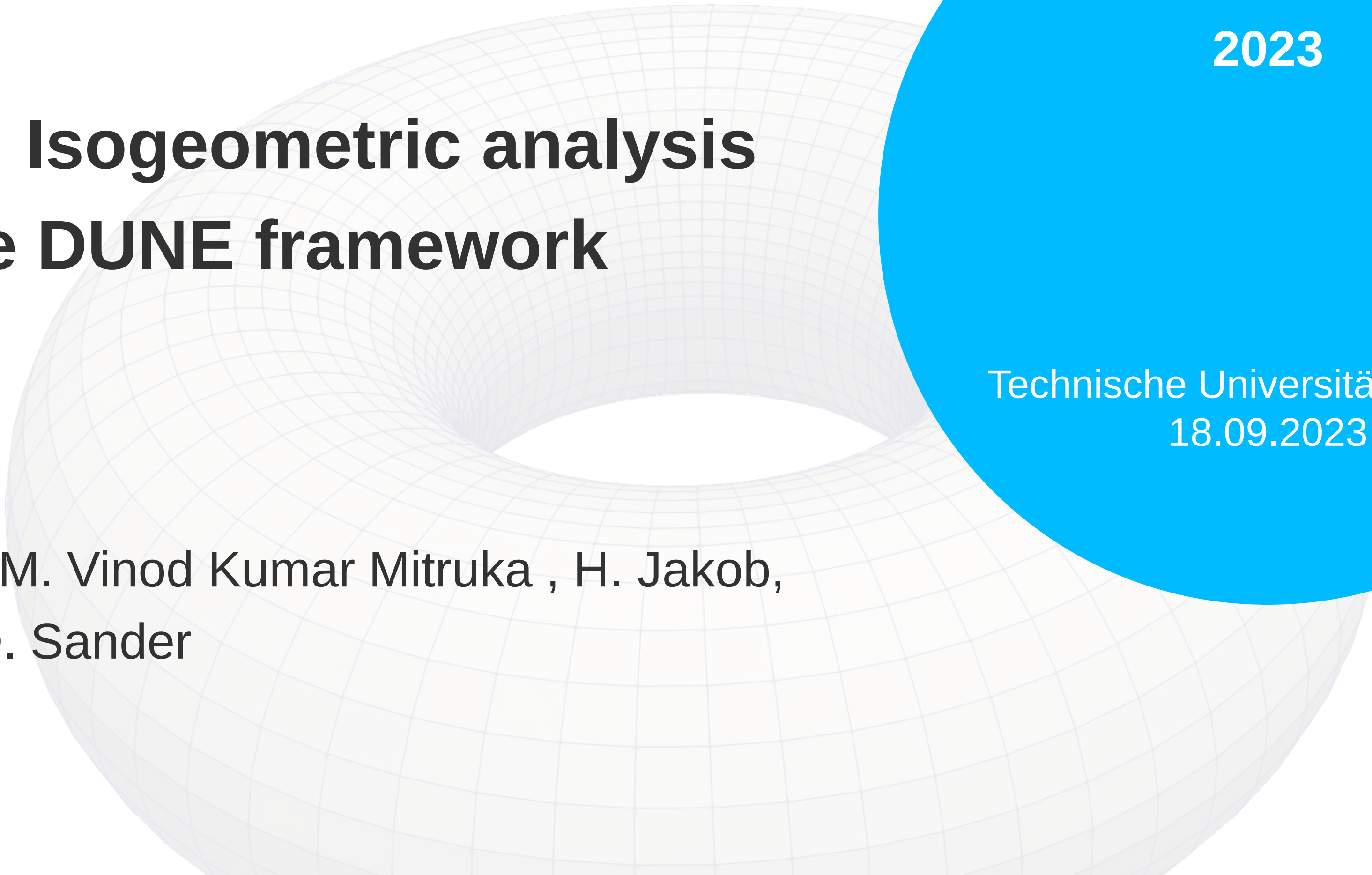
University of Stuttgart
Institute for Structural Mechanics



Dune User Meeting 2023

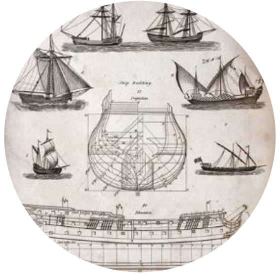
dune-iga: Isogeometric analysis within the DUNE framework

Technische Universität Dresden
18.09.2023

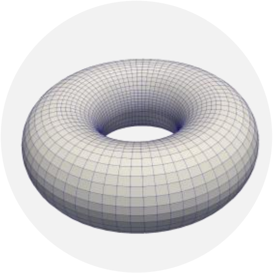
A. Müller, T.K.M. Vinod Kumar Mitruka , H. Jakob,
M. Bischoff, O. Sander

Baustatik und Baudynamik

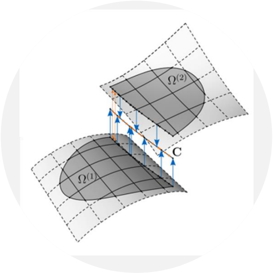
Outline



NURBS and Isogeometric Analysis



dune-iga

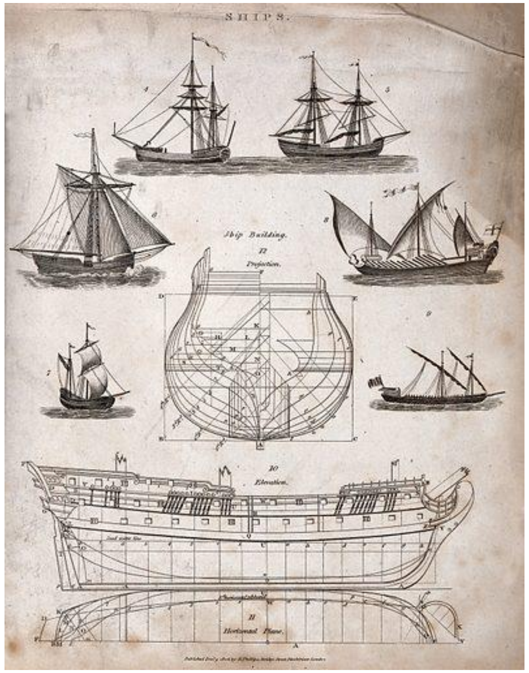


Challenges and Summary

NURBS
and
Isogeometric Analysis

NURBS and Isogeometric Analysis

History



Ship-building: six kinds of ship (top), the hull of a ship of the line (centre and below). Engraving, 1806. Created 9 November 1806. Ships. Shipbuilding. License: (CC BY 4.0)

https://commons.wikimedia.org/wiki/File:Krzywiki.jpg

Elsevier

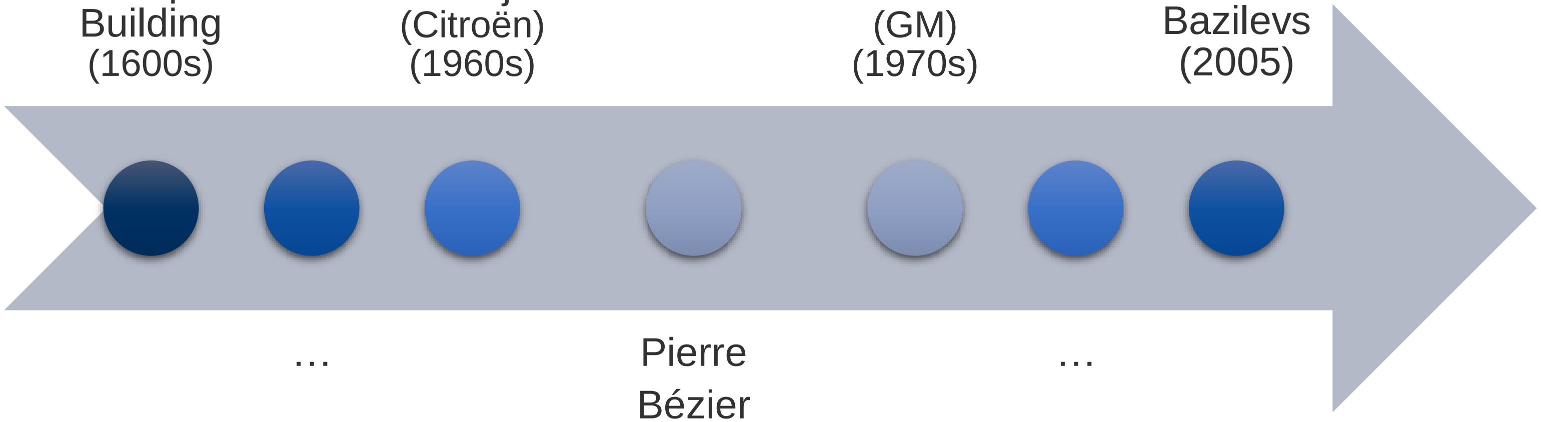
Wikipedia

Ship Building (1600s)

Paul de Casteljau (Citroën) (1960s)

Carl de Boor (GM) (1970s)

Hughes, Cottrell, Bazilevs (2005)



…

Pierre Bézier (Renault) (1960s)

…

Wikipedia

Sergei Bernstein

Wikipedia

utexas.edu

Alastair Townsend: On the Spline: A Brief History of the Computational Curve (Full) - https://alatown.com/spline-history-architecture/

Gallo and Wirz 2021: The evolution of the digital curve: from shipbuilding spline to the diffusion of NURBS subdivision surface and T-splines as tools in architectural design

Basis Functions, Continuity, and Patches

B-spline basis functions:

Cox-de Boor recursion formula

$p = 0$:

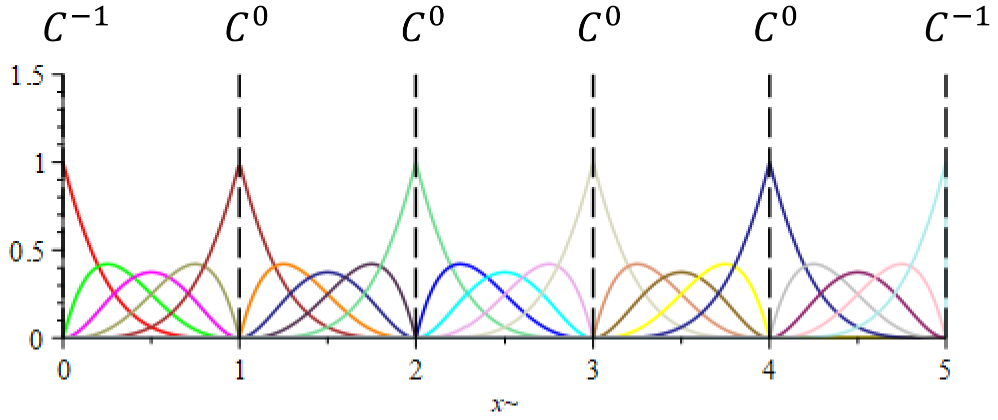
$$N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \leq \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

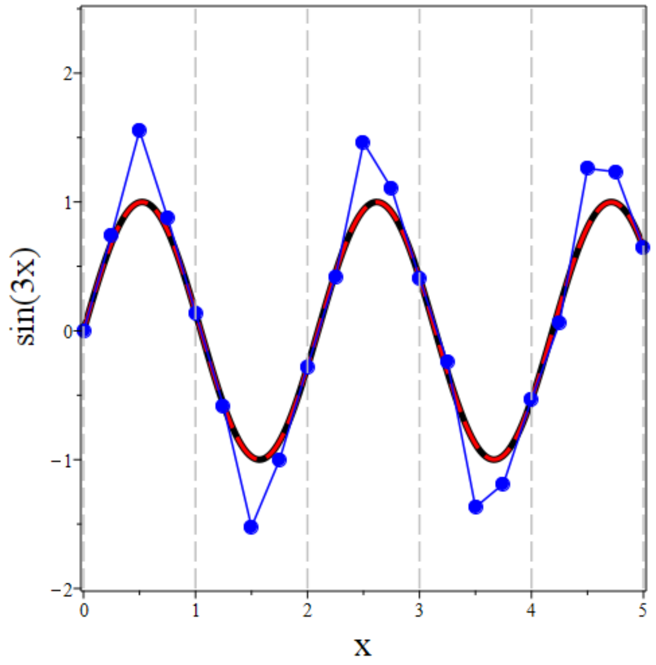
$p = 1, 2, 3, \ldots$:

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi)$$

$$\boldsymbol{\Xi} = [0,0,0,0,0,1,1,1,1,2,2,2,2,3,3,3,3,4,4,4,4,5,5,5,5,5]$$

$p = 4$





Curve: $\mathbf{C}(\xi) = \sum_{i=1}^{n} N_{i,p}(\xi)\mathbf{B}_i$

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Basis Functions, Continuity, and Patches

B-spline basis functions:

Cox-de Boor recursion formula

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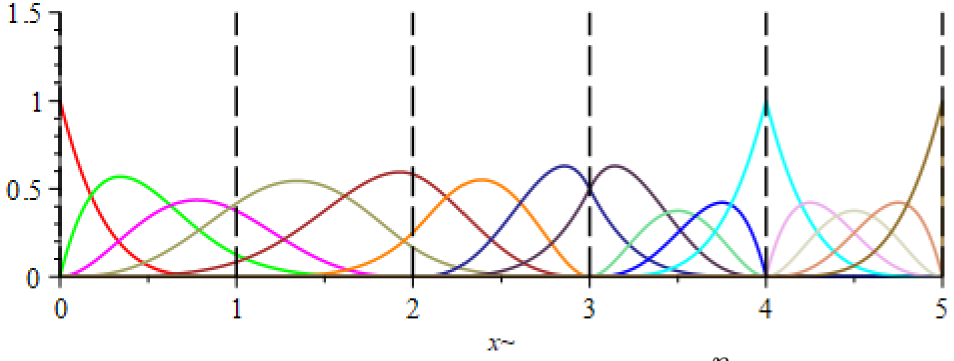
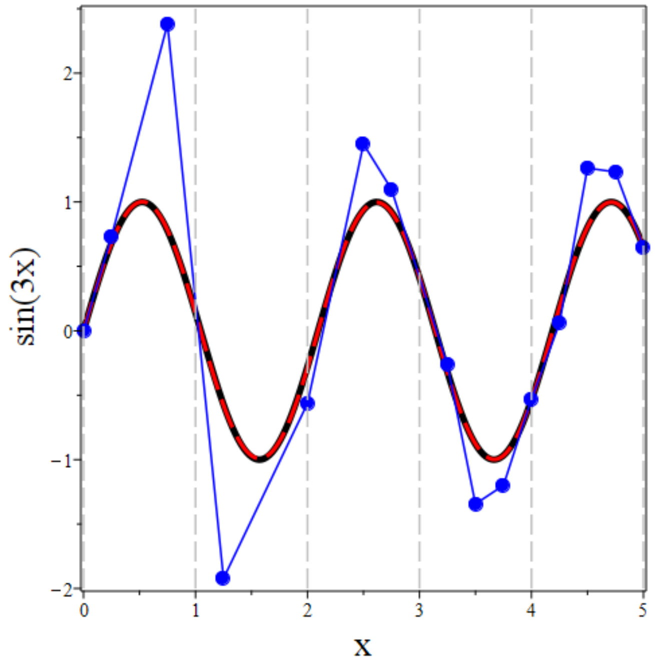
$$\boldsymbol{\Xi} = [0, 0, 0, 0, 0, 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5]$$

C^{-1} C^{3} C^{2} C^{1} C^{0} C^{-1}

$p = 4$



Curve: $\mathbf{C}(\xi) = \sum_{i=1}^{n} N_{i,p}(\xi) \mathbf{B}_i$



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NURBS and Isogeometric Analysis

Basis Functions, Continuity, and Patches

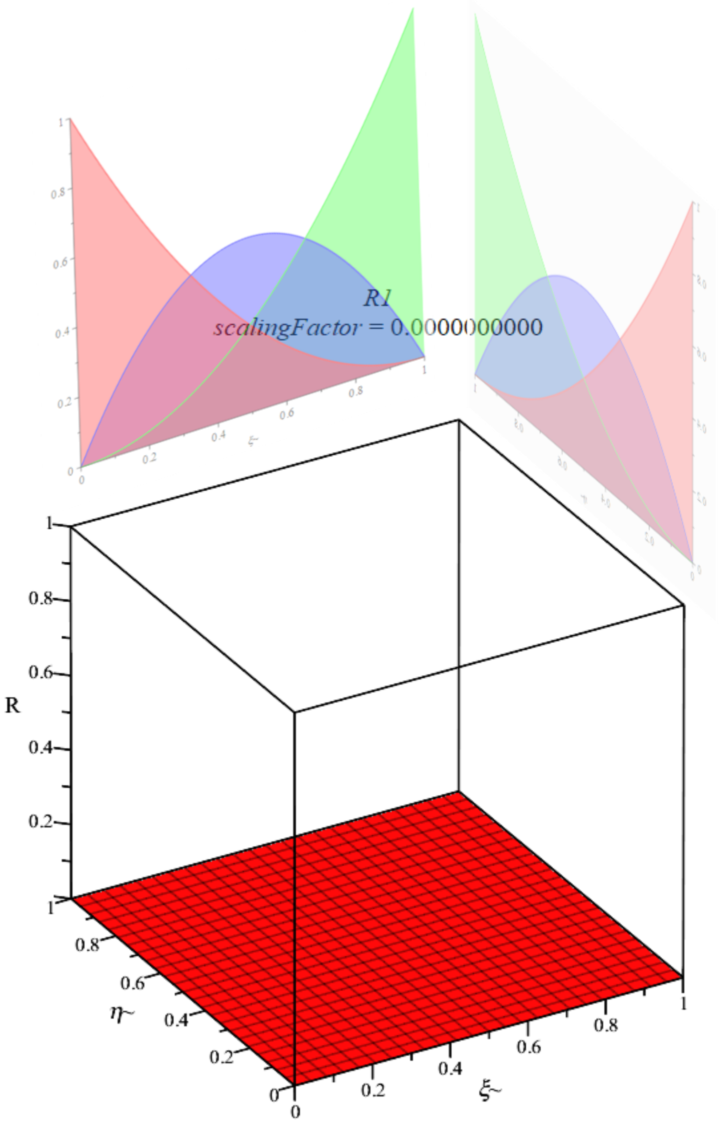
Non-**U**niform **R**ational **B**-**S**plines (NURBS):

$$R_i^p(\xi) = \frac{N_{i,p}(\xi)\, w_i}{\sum_{\hat{i}=1}^{n} N_{\hat{i},p}(\xi)\, w_{\hat{i}}}$$

$$\mathbf{C}(\xi) = \sum_{i=1}^{n} R_i^p(\xi)\mathbf{B}_i$$

Surfaces: (Tensor product)

$$R_{i,j}^{p,q}(\xi,\eta) = \frac{N_{i,p}(\xi) M_{j,q}(\eta)\, w_{i,j}}{\sum_{\hat{i}=1}^{n} \sum_{\hat{j}=1}^{m} N_{\hat{i},p}(\xi) M_{\hat{j},q}(\eta)\, w_{\hat{i},\hat{j}}}$$



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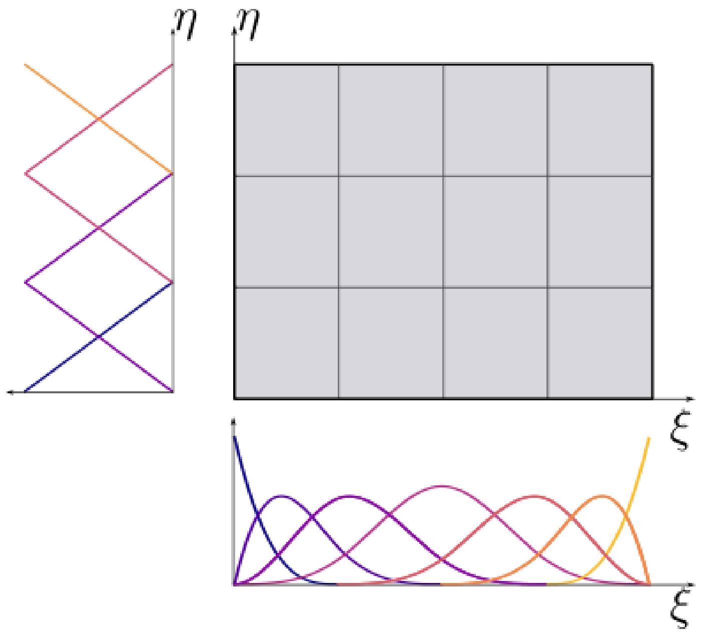
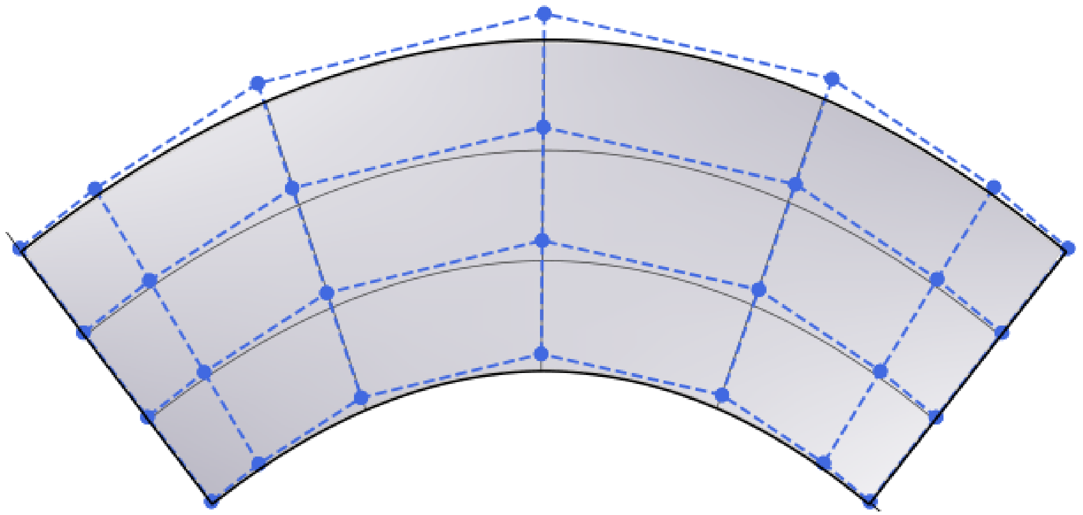
NURBS and Isogeometric Analysis

Basis Functions, Continuity, and Patches



(a) surface in the parameter space

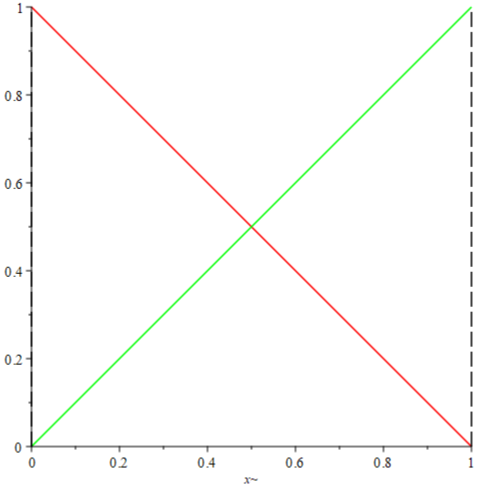


(b) surface in the physical space

NURBS and Isogeometric Analysis

Baustatik und Baudynamik

h-, p-, and k-refinement



$$\Xi = [0, 0, 1, 1]$$

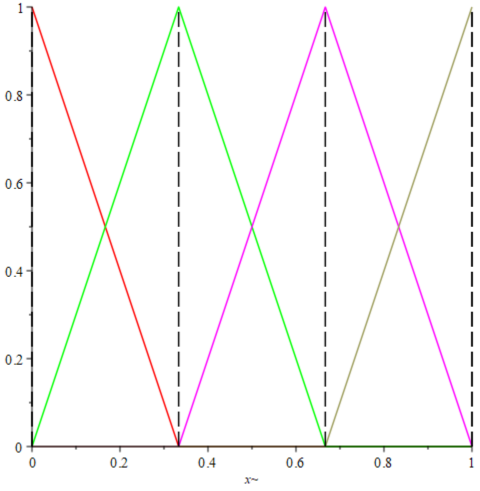
h-refinement

$p = 1$

$n = 2$

$p = 1$

$n = 4$

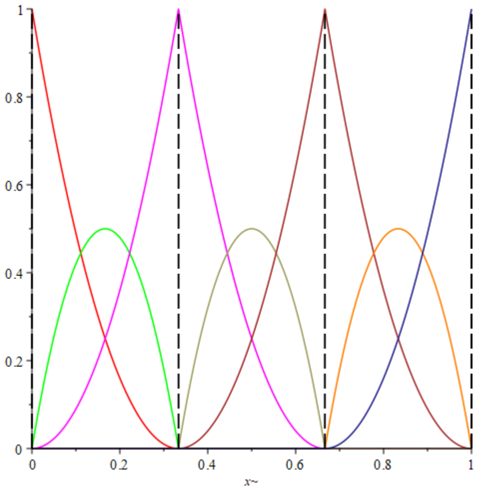


$$\Xi = \left[0, 0, \tfrac{1}{3}, \tfrac{2}{3}, 1, 1\right]$$

p-refinement

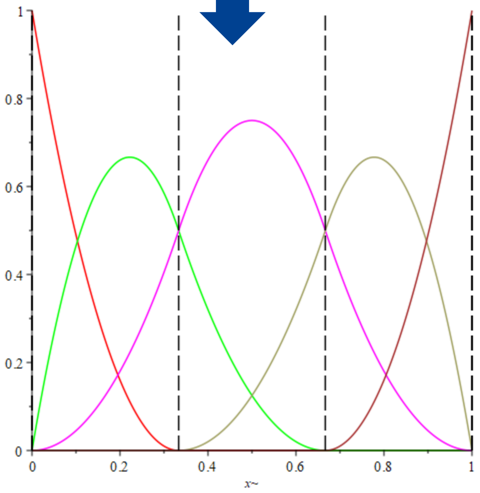
$p = 2$

$n = 7$



$$\Xi = \left[0, 0, 0, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{2}{3}, \tfrac{2}{3}, 1, 1, 1\right]$$

k-refinement



$$\Xi = \left[0, 0, 0, \tfrac{1}{3}, \tfrac{2}{3}, 1, 1, 1\right]$$

$p = 2$

$n = 5$

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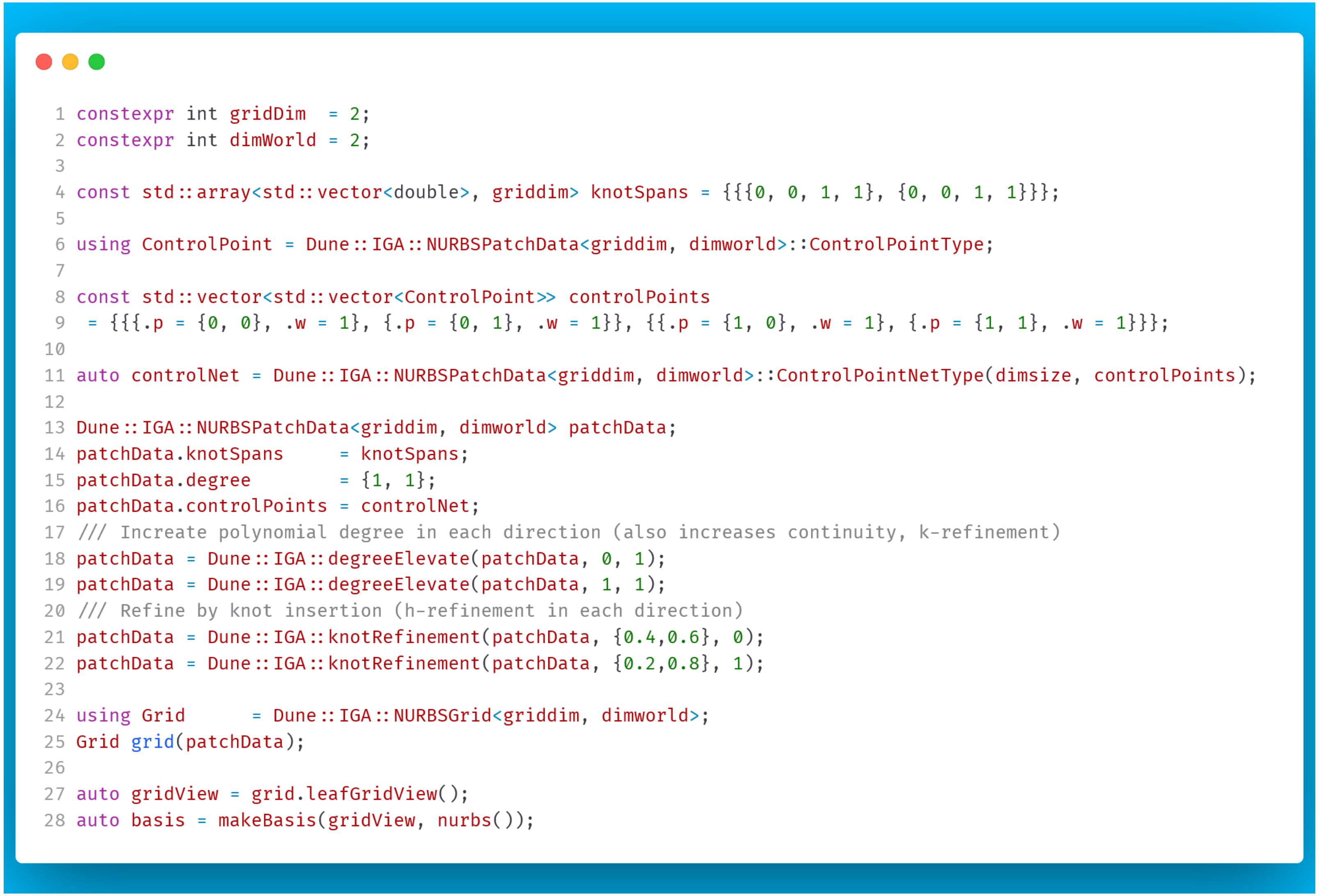
dune-iga

dune-iga

Interface (Example)



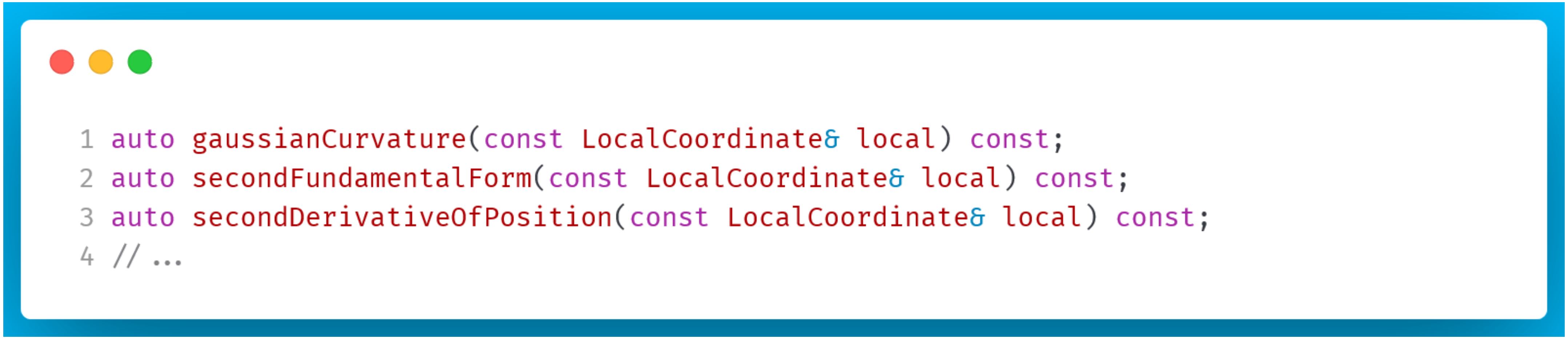
```cpp
constexpr int gridDim  = 2;
constexpr int dimWorld = 2;

const std::array<std::vector<double>, griddim> knotSpans = {{{0, 0, 1, 1}, {0, 0, 1, 1}}};

using ControlPoint = Dune::IGA::NURBSPatchData<griddim, dimworld>::ControlPointType;

const std::vector<std::vector<ControlPoint>> controlPoints
 = {{{.p = {0, 0}, .w = 1}, {.p = {0, 1}, .w = 1}}, {{.p = {1, 0}, .w = 1}, {.p = {1, 1}, .w = 1}}};

auto controlNet = Dune::IGA::NURBSPatchData<griddim, dimworld>::ControlPointNetType(dimsize, controlPoints);

Dune::IGA::NURBSPatchData<griddim, dimworld> patchData;
patchData.knotSpans     = knotSpans;
patchData.degree        = {1, 1};
patchData.controlPoints = controlNet;
/// Increate polynomial degree in each direction (also increases continuity, k-refinement)
patchData = Dune::IGA::degreeElevate(patchData, 0, 1);
patchData = Dune::IGA::degreeElevate(patchData, 1, 1);
/// Refine by knot insertion (h-refinement in each direction)
patchData = Dune::IGA::knotRefinement(patchData, {0.4,0.6}, 0);
patchData = Dune::IGA::knotRefinement(patchData, {0.2,0.8}, 1);

using Grid      = Dune::IGA::NURBSGrid<griddim, dimworld>;
Grid grid(patchData);

auto gridView = grid.leafGridView();
auto basis = makeBasis(gridView, nurbs());
```

dune-iga

Additional features

- Create another Grid from NURBS geometry (PR)
- Python Bindings
- …

Geometry of a grid entity



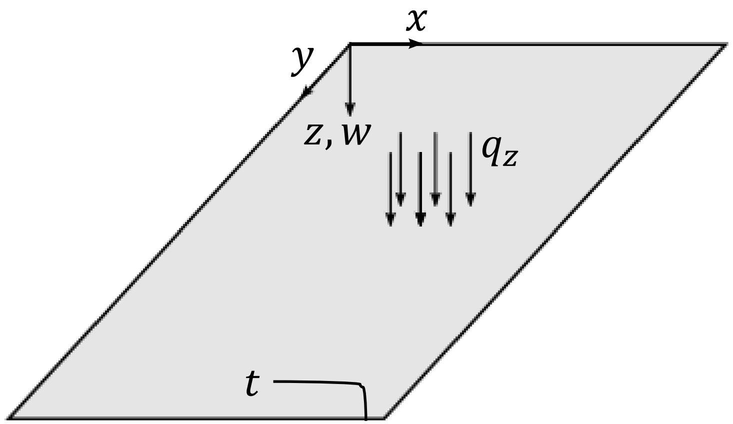
```
auto gaussianCurvature(const LocalCoordinate& local) const;
auto secondFundamentalForm(const LocalCoordinate& local) const;
auto secondDerivativeOfPosition(const LocalCoordinate& local) const;
// ...
```

Example 1 – Kirchhoff Plate

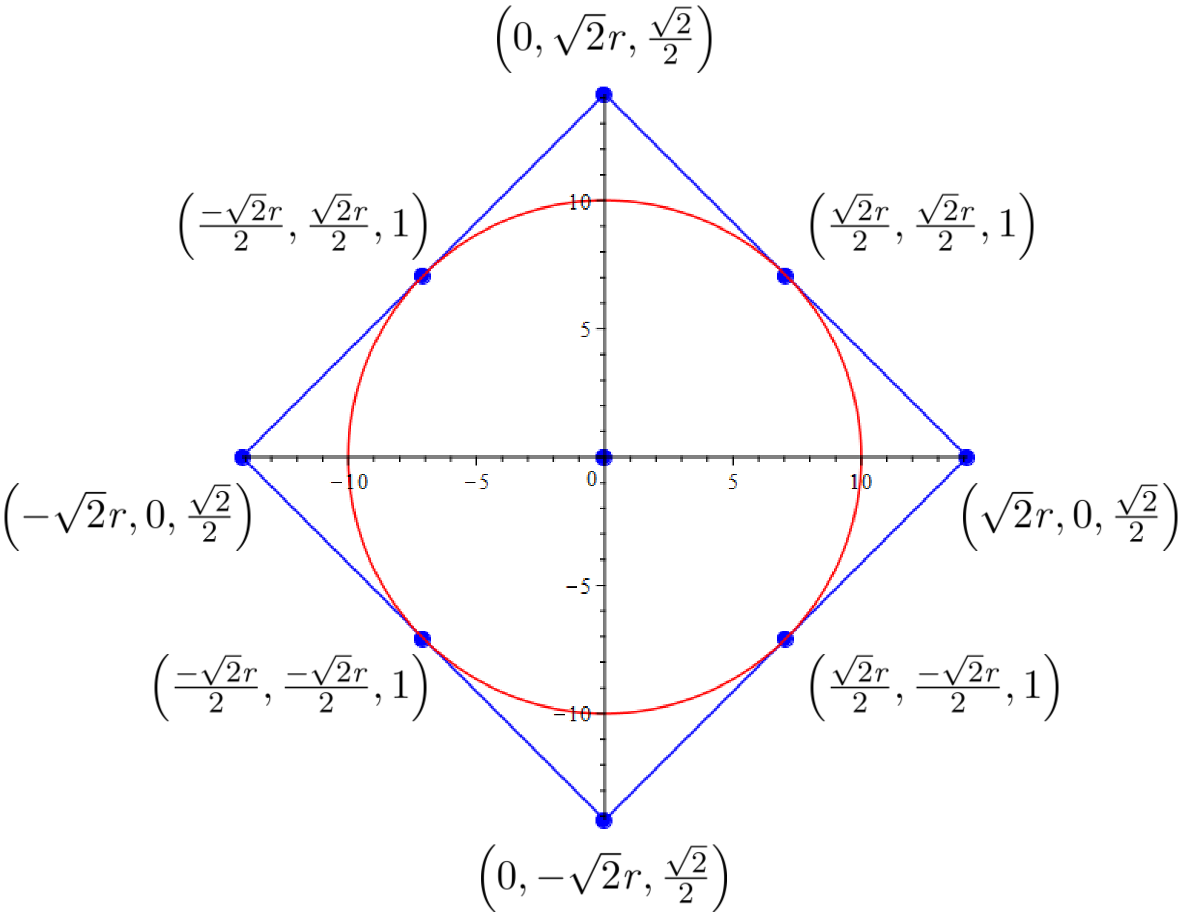
PDE:

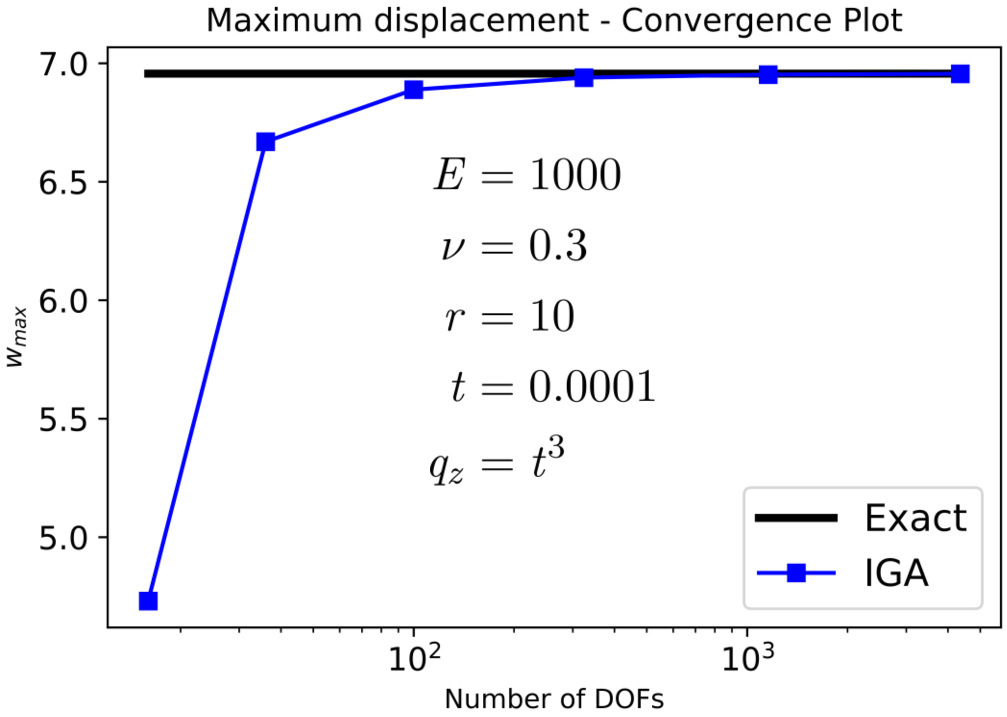
$$\nabla^2\nabla^2 w = \frac{q_z}{D} \quad ; \quad D = \frac{Et^3}{12(1-\nu^2)}$$

Weak form needs C^1-continuity

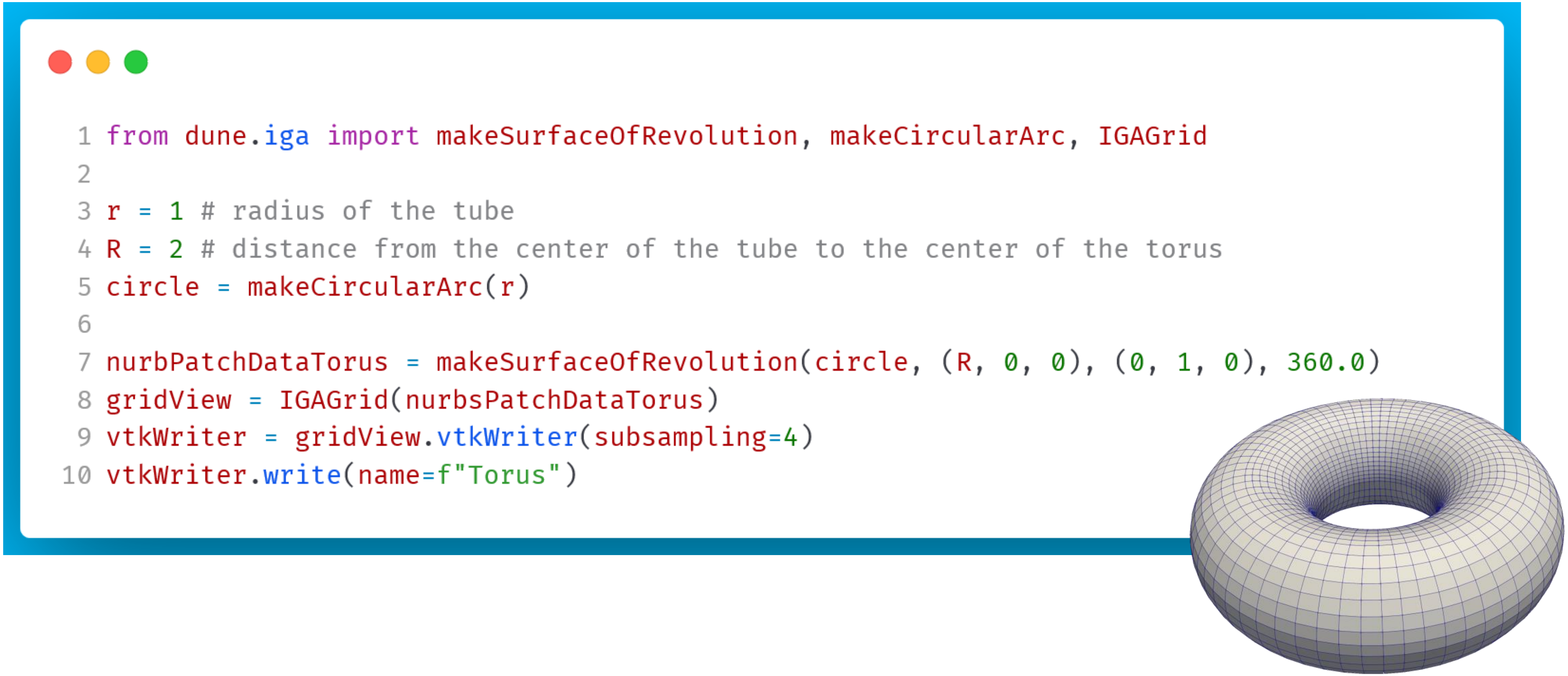


Example 1 – Kirchhoff plate theory – simply supported circular plate with distributed load


$\left(0, \sqrt{2}r, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{-\sqrt{2}r}{2}, \frac{\sqrt{2}r}{2}, 1\right)$
$\left(\frac{\sqrt{2}r}{2}, \frac{\sqrt{2}r}{2}, 1\right)$
$\left(-\sqrt{2}r, 0, \frac{\sqrt{2}}{2}\right)$
$\left(\sqrt{2}r, 0, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{-\sqrt{2}r}{2}, \frac{-\sqrt{2}r}{2}, 1\right)$
$\left(\frac{\sqrt{2}r}{2}, \frac{-\sqrt{2}r}{2}, 1\right)$
$\left(0, -\sqrt{2}r, \frac{\sqrt{2}}{2}\right)$
10
5
−10
−5
0
5
10
−5
−10



Example 2 – Python bindings

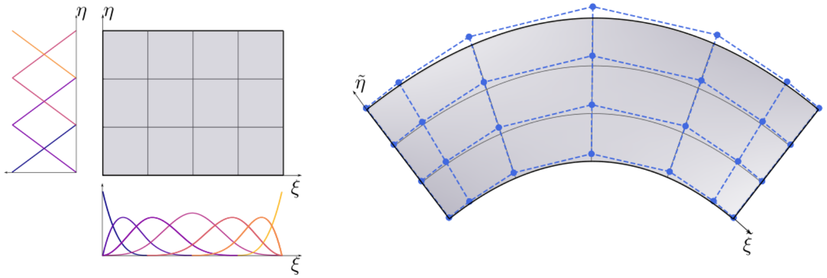


```python
from dune.iga import makeSurfaceOfRevolution, makeCircularArc, IGAGrid

r = 1 # radius of the tube
R = 2 # distance from the center of the tube to the center of the torus
circle = makeCircularArc(r)

nurbPatchDataTorus = makeSurfaceOfRevolution(circle, (R, 0, 0), (0, 1, 0), 360.0)
gridView = IGAGrid(nurbsPatchDataTorus)
vtkWriter = gridView.vtkWriter(subsampling=4)
vtkWriter.write(name=f"Torus")
```

dune-iga

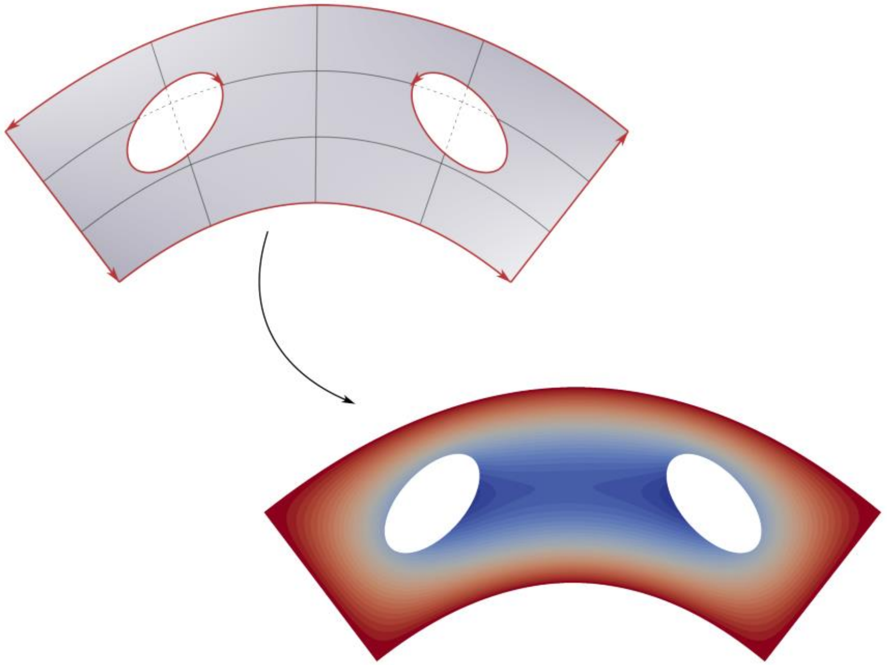
Trimmed NURBS

- Tensor product nature restricts possible geometries
- Trimmed NURBS (Typical CAD representation)

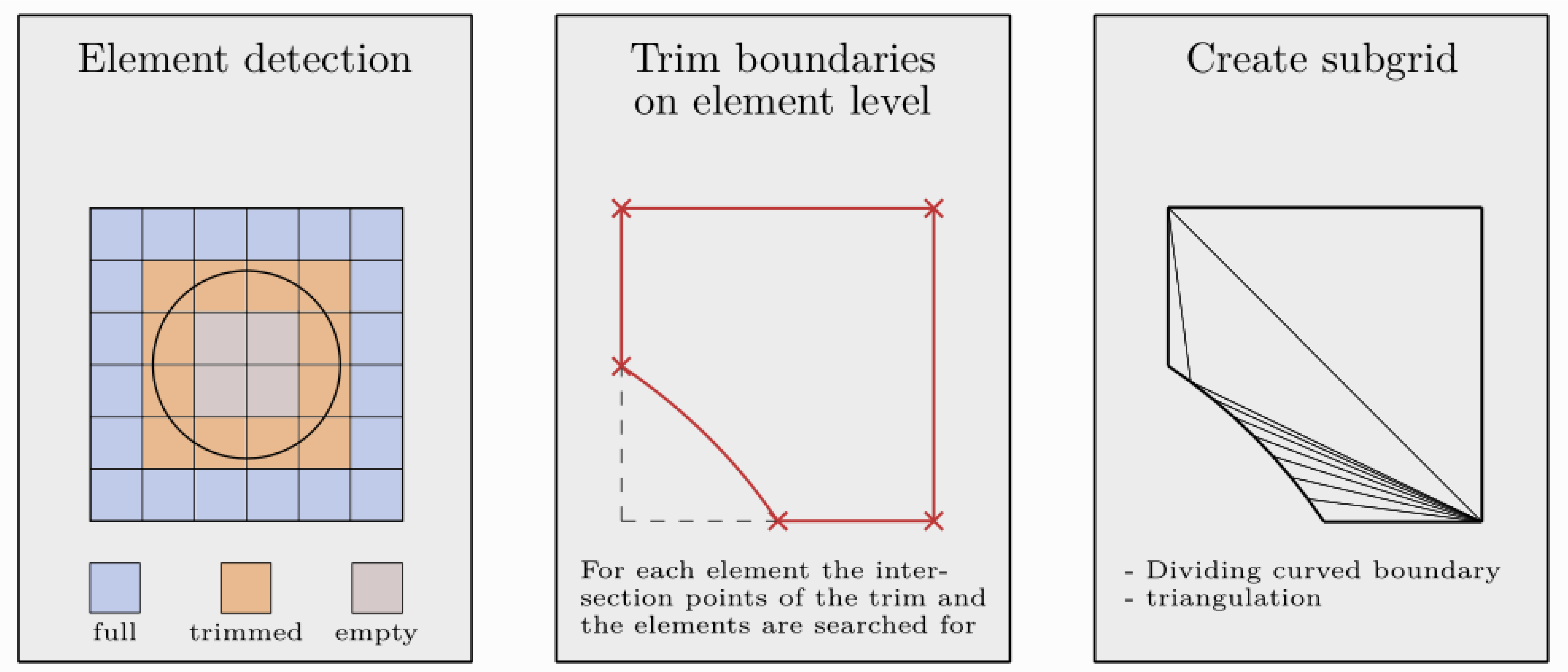


(a) surface in the parameter space

(b) surface in the physical space



Trimmed NURBS



Element detection

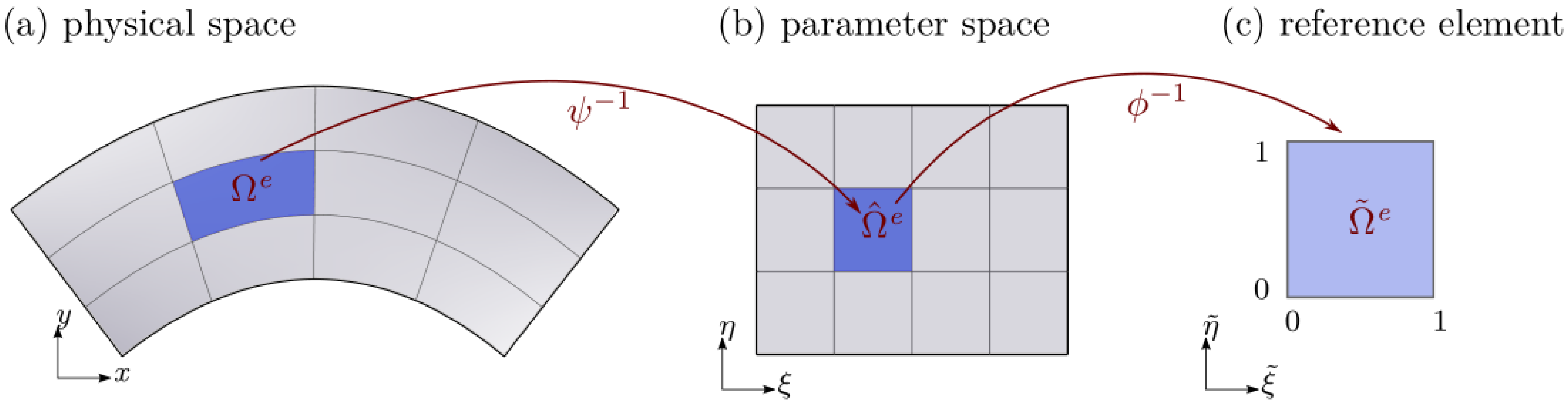
Trim boundaries on element level

For each element the intersection points of the trim and the elements are searched for

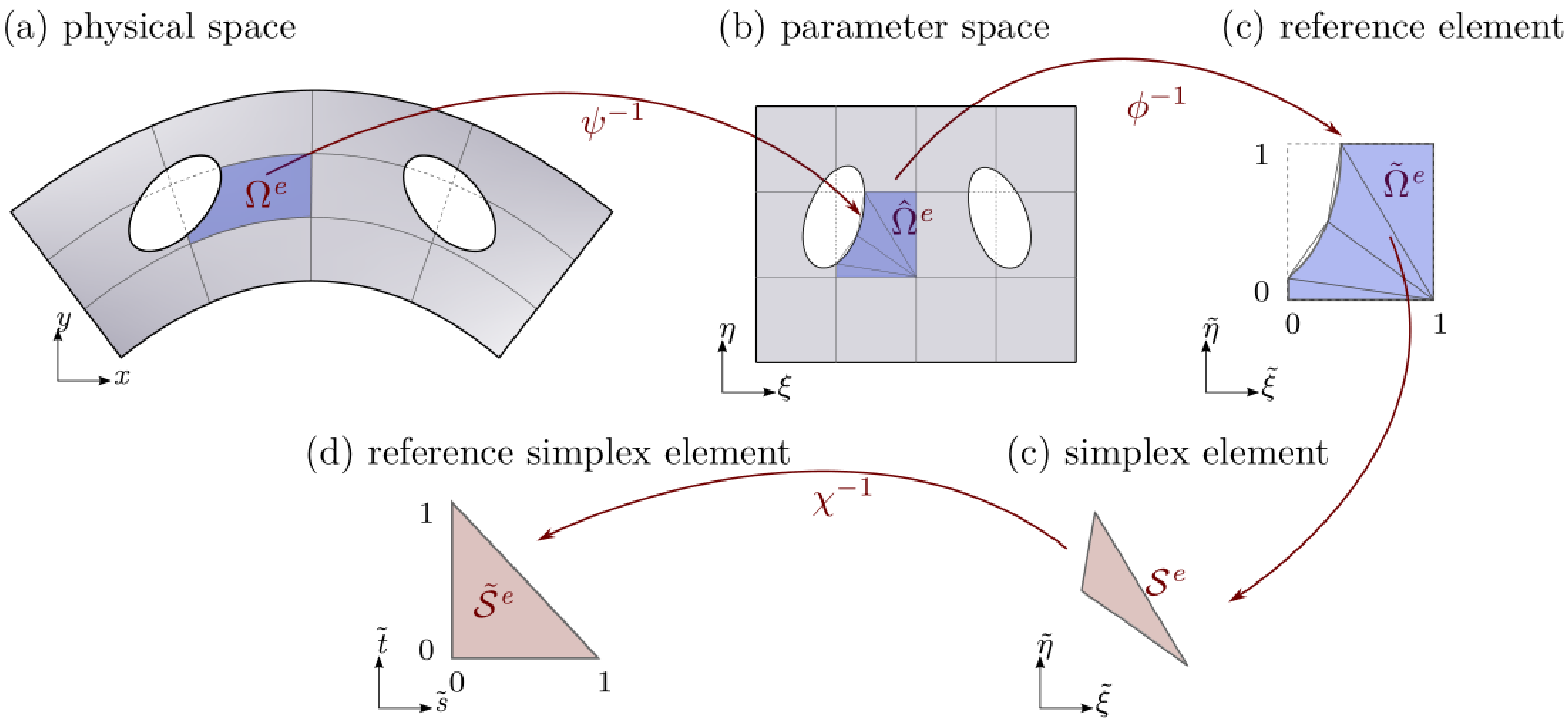
Create subgrid

- Dividing curved boundary
- triangulation

Untrimmed NURBS

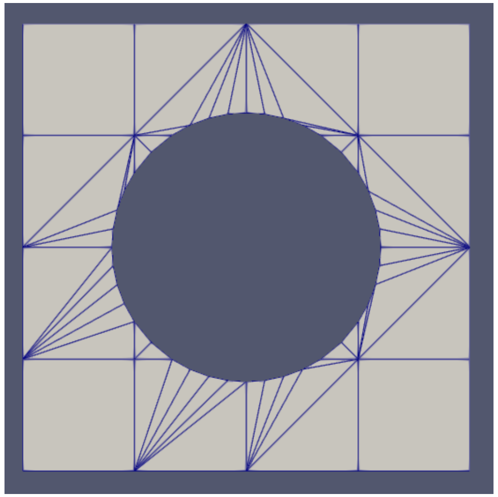


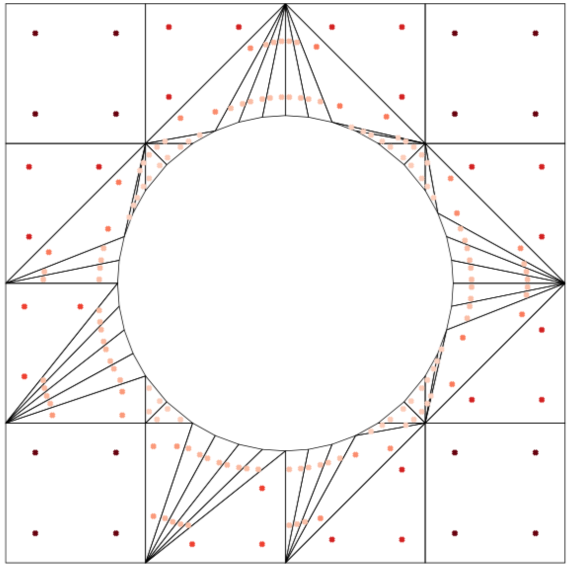
Trimmed NURBS

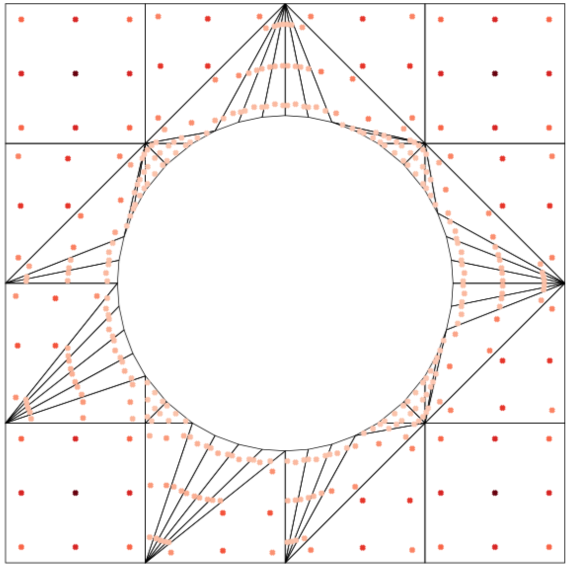


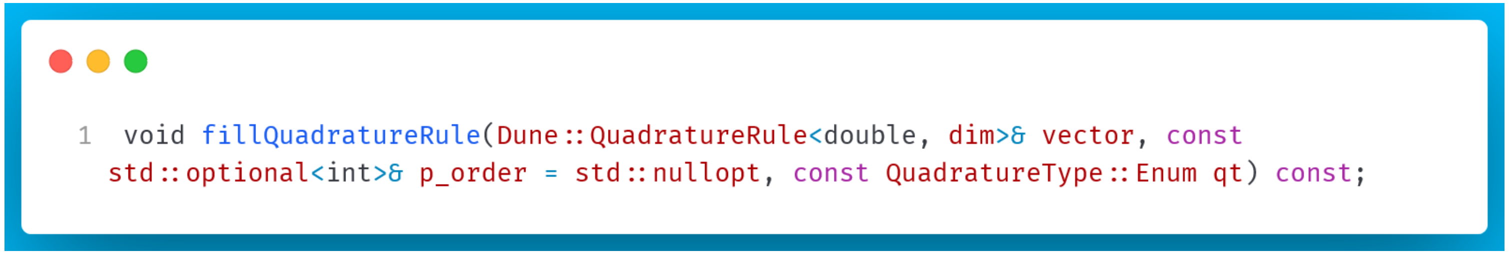
dune-iga

Trimmed NURBS - Interface









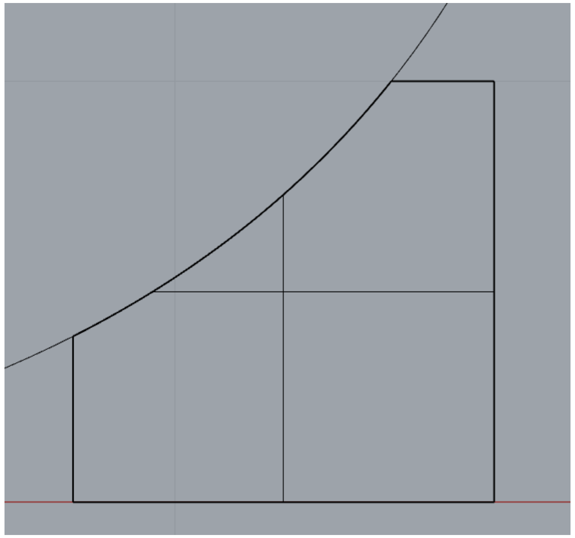
```
void fillQuadratureRule(Dune::QuadratureRule<double, dim>& vector, const
std::optional<int>& p_order = std::nullopt, const QuadratureType::Enum qt) const;
```

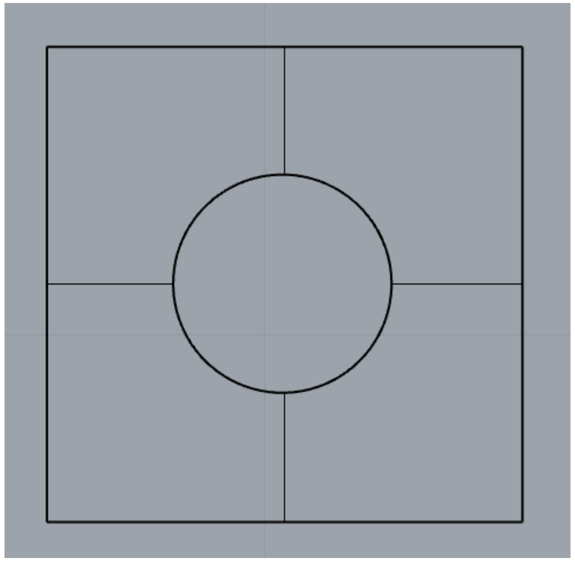
dune-iga

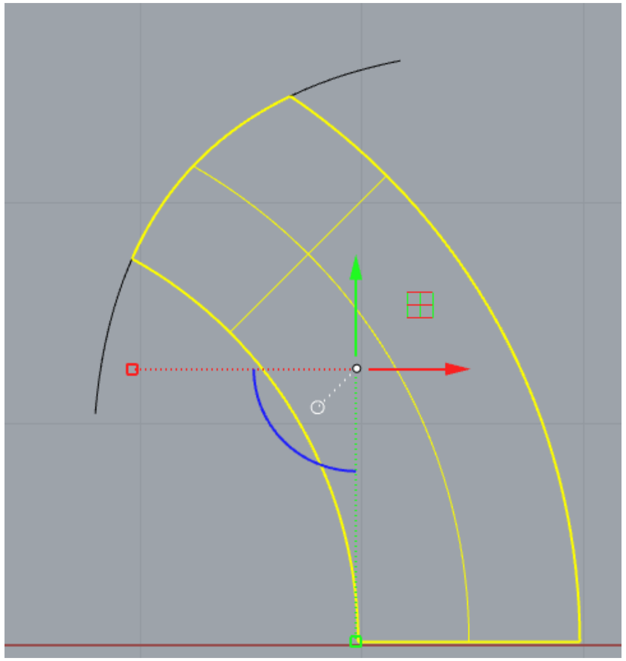
Trimmed NURBS

Import possible from RHINO

Rhinoceros®





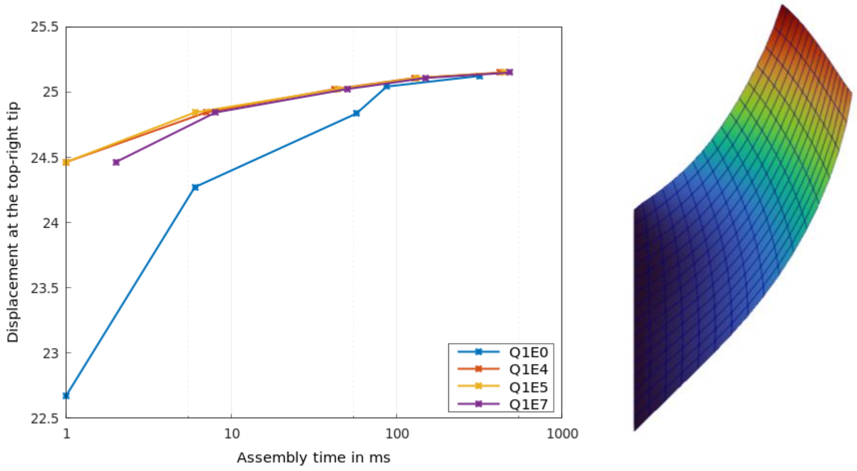


Interlude

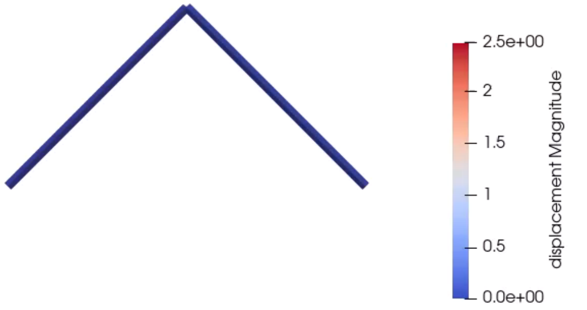
Ikarus – a DUNE module

- Our in-house DUNE front-end
- Global assemblers
- local assembler expression templates (`dune-localfefunctions`)
- Some non-linear solvers, …

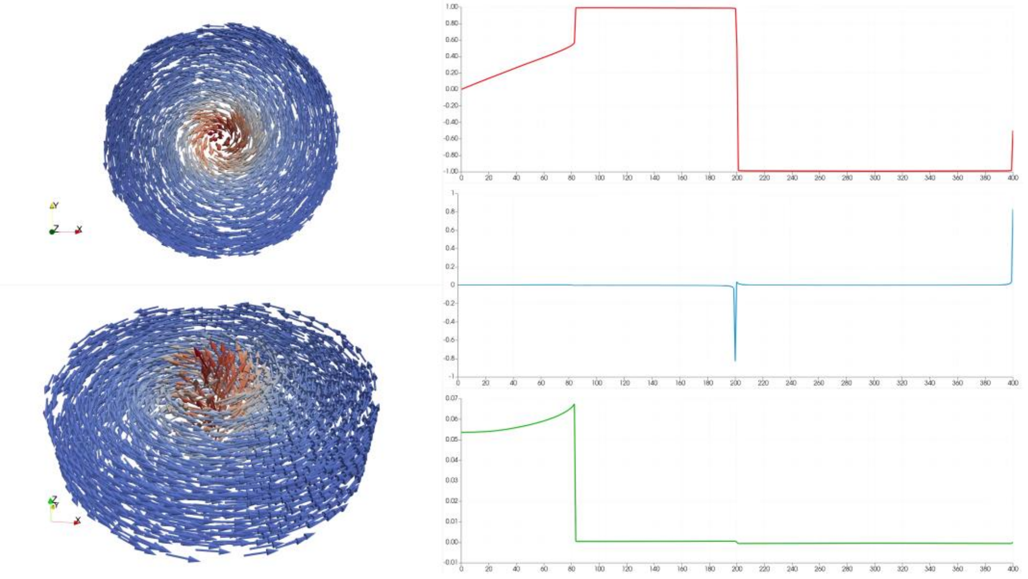
Linear elasticity



Non-linear Von-Mises Truss



ikarus-project.github.io



Micro-magnetics

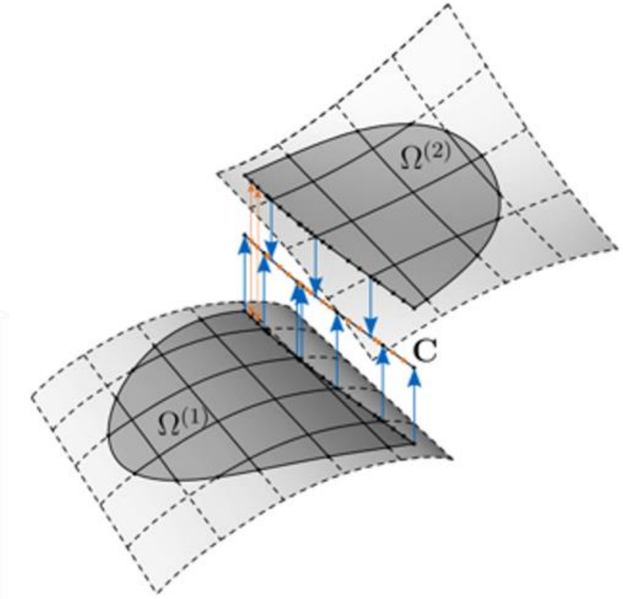
Outlook

Outlook

- Generalize notion of reference Elements dune-geometry_Issue32

  (`GeometryType::none` is not always checked)

- Add hierarchic grid features (only `leafGridView()` available)
- Multiple Patches
- Interface for higher derivative transformations?



Similar interface as for grid element intersections?

TESCHEMACHER 2023

Universität Stuttgart

ikarus-project.github.io

Vielen Dank!

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