On the Analysis of Porous Media Dynamics using a DUNE-PANDAS Interface

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Outline

- Introduction
- Theory of Porous Media
- Numerical Treatment
- Interface
- Numerical Example
- Summary & Outlook

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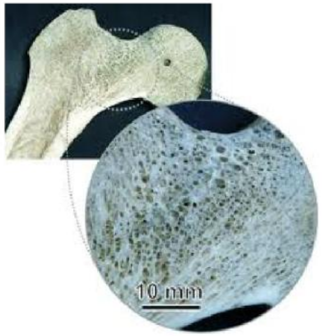
Introduction

➢ Porous media can be found in different branches of engineering

foams in mechanical engineering

soil and concrete in civil engineering



soft and hard tissue in biological engineering

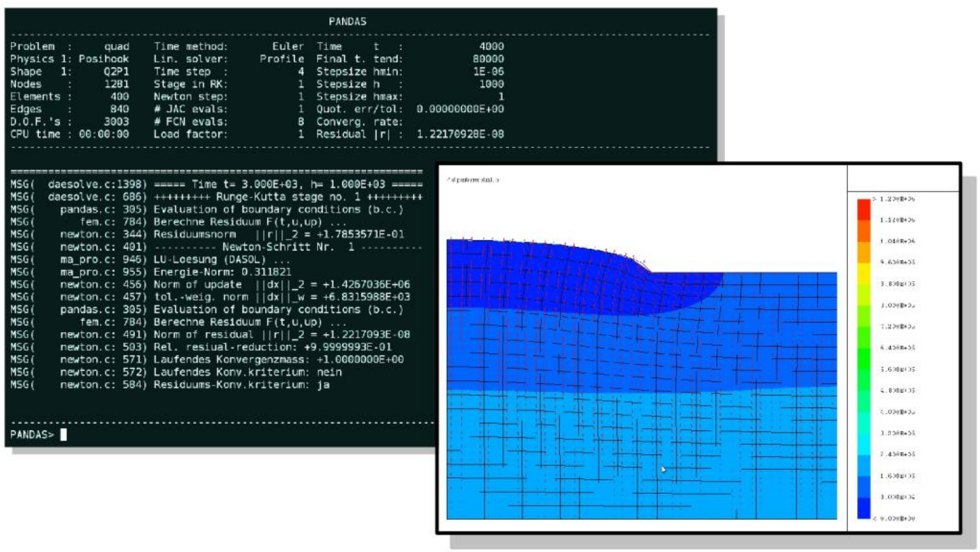
➢ Solid skeleton can undergo large deformations

➢ Goal:

- Simulating the interaction between the deformable solid skeleton and the pore fluid in large-scale problems

➢ Method:

- Description of the problem by the Theory of Porous Media (TPM)
- Discretisation by the Finite Element Method within the framework of PANDAS[1]



⟹ as a purely sequential code, PANDAS has limited capabilities

⟹ parallelisation of PANDAS by use of a DUNE-PANDAS interface

[1] **P**orous media **A**daptive **N**on-linear finite element solver based on **D**ifferential **A**lgebraic **S**ystems

PANDAS COUPLED FEM SOLVER
www.get-pandas.com

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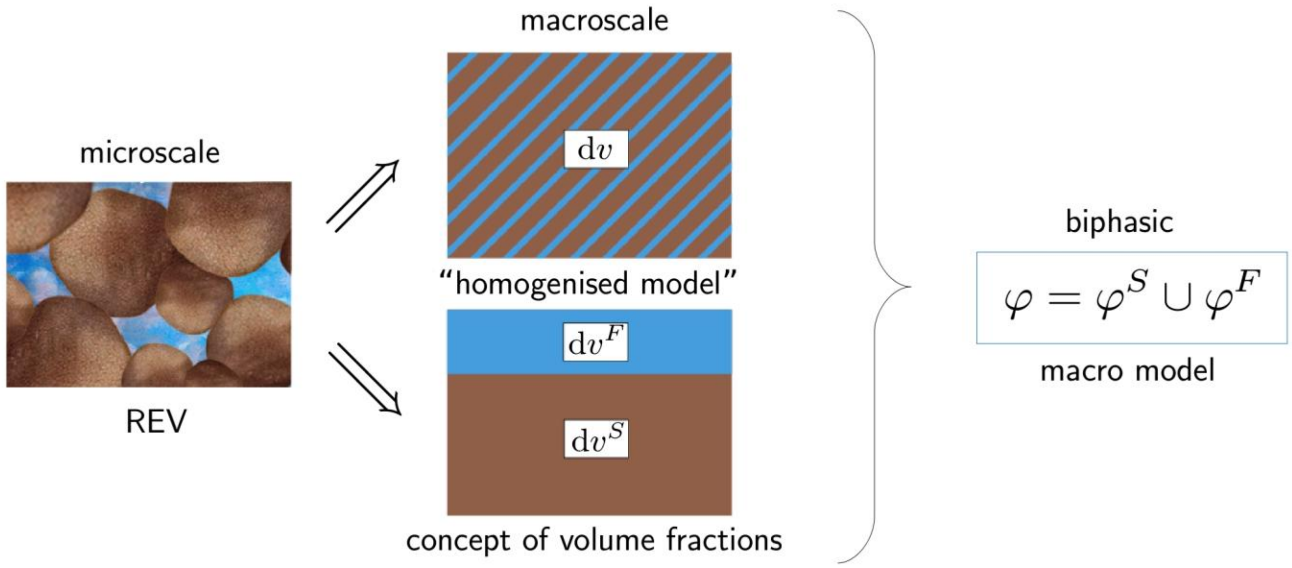
Theory of Porous Media (TPM)

➢ **Homogenisation:** micro-to-macro transition [*Bowen, de Boer, Ehlers*]

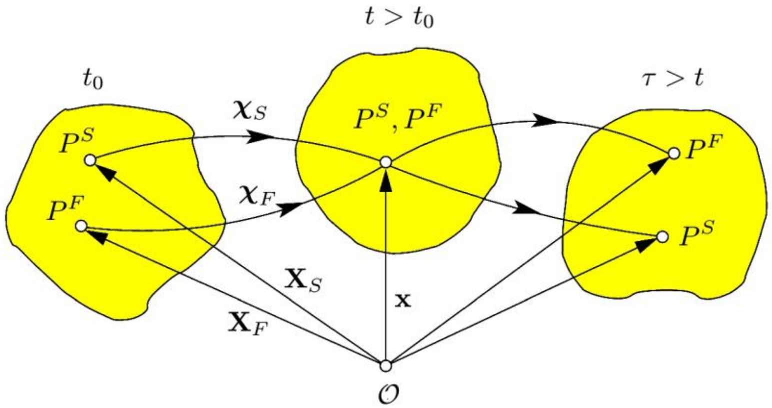


- **Concept of volume fractions:** $$\sum_\alpha n^\alpha = \sum_\alpha \frac{\mathrm{d}v^\alpha}{\mathrm{d}v} = 1 \quad \text{with} \quad \alpha = \{S, F\}$$

- **Density relations:** $$\left.\begin{aligned} \rho^{\alpha R} &= \frac{\mathrm{d}m^\alpha}{\mathrm{d}v^\alpha} \\ \rho^\alpha &= \frac{\mathrm{d}m^\alpha}{\mathrm{d}v} \end{aligned}\right\} \quad \rho^\alpha = n^\alpha \rho^{\alpha R}$$

Mixture Kinematics

- **Superimposed continua with individual states of motion**



$$\mathbf{x} = \boldsymbol{\chi}_\alpha(\mathbf{X}, t)$$

$$\Longrightarrow \quad \overset{\prime}{\mathbf{x}}_\alpha = \frac{\mathrm{d}\boldsymbol{\chi}_\alpha(\mathbf{X}_\alpha, t)}{\mathrm{d}t}, \qquad \overset{\prime\prime}{\mathbf{x}}_\alpha = \frac{\mathrm{d}^2\boldsymbol{\chi}_\alpha(\mathbf{X}_\alpha, t)}{\mathrm{d}t^2}$$

- **Motion of the solid skeleton (*Lagrange*an)**

Displacement: $\mathbf{u}_S = \mathbf{x} - \mathbf{X}_S$

Velocity: $\mathbf{v}_S = \overset{\prime}{\mathbf{x}}_S = (\mathbf{u}_S)'_S$

Acceleration: $(\mathbf{v}_S)'_S = \overset{\prime\prime}{\mathbf{x}}_S = (\mathbf{u}_S)''_S$

- **Motion of the pore fluid (modified *Euler*ian)**

Velocity: $\mathbf{v}_F = \overset{\prime}{\mathbf{x}}_F$

Acceleration: $(\mathbf{v}_F)'_F = (\mathbf{v}_F)'_S + \operatorname{grad}\mathbf{v}_F\,(\mathbf{v}_F - \mathbf{v}_S)$

Seepage velocity: $\mathbf{w}_F = \mathbf{v}_F - \mathbf{v}_S$

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➢ Incompressible binary porous media model

- **Assumptions:**
  - ∗ binary model
    $\alpha = \{S, F\}$
  - ∗ no mass production
    $\hat{\rho}^S = \hat{\rho}^F = 0$
  - ∗ incompressible constituents
    $(\rho^{SR})'_S = (\rho^{FR})'_S = 0$
  - ∗ equal body forces (gravitation)
    $\mathbf{b}^S = \mathbf{b}^F = \mathbf{b}$
  - ∗ fully saturated soil
    $n^S + n^F = 1$
  - ∗ iso-thermal conditions
    $\Theta^S = \Theta^F = \Theta = \text{const.}$

- **Incompressible binary porous media balance equations**
  - ∗ total momentum balance (solid and pore fluid):

$$\rho^S (\mathbf{v}_S)'_S + \rho^F [(\mathbf{v}_F)'_S + (\operatorname{grad} \mathbf{v}_F)\mathbf{w}_F] = \operatorname{div}(\mathbf{T}^S + \mathbf{T}^F) + (\rho^S + \rho^F)\mathbf{b} + \underbrace{\hat{\mathbf{s}}^S + \hat{\mathbf{s}}^F}_{=\mathbf{0}}$$

  - ∗ fluid momentum balance:

$$\rho^F (\mathbf{v}_F)'_S + \rho^F (\operatorname{grad} \mathbf{v}_F)\mathbf{w}_F = \operatorname{div} \mathbf{T}^F + \rho^F \mathbf{b} + \hat{\mathbf{s}}^F$$

  - ∗ total volume balance (solid and pore fluid):

$$0 = \operatorname{div}[(\mathbf{u}_S)'_S + n^F \mathbf{w}_F]$$

where:

$\mathbf{T}^S/\mathbf{T}^F$	:	partial Cauchy stress tensors (solid/fluid)
$\hat{\mathbf{s}}^S/\hat{\mathbf{s}}^F$	:	momentum productions (solid/fluid), with $\hat{\mathbf{s}}^\alpha = \hat{\mathbf{p}}^\alpha + \hat{\rho}^\alpha \overset{\prime}{\mathbf{x}}_\alpha$
$\mathbf{b}$	:	body force

Constitutive Setting

- **Principle of effective stress** [*Therzaghi* 1925; *Skempton* 1960; *Ehlers* 1993]

$$\begin{aligned} \mathbf{T}^S &= \mathbf{T}^S_E - n^S p\,\mathbf{I} \\ \mathbf{T}^F &= \mathbf{T}^F_E - n^F p\,\mathbf{I} \\ \hat{\mathbf{p}}^F &= \hat{\mathbf{p}}^F_E + p\,\mathrm{grad}\,n^F \end{aligned}$$

with: $\mathbf{T}^F_E \approx \mathbf{0}$

- **Fluid constituent** (isotropic, lingering fluid condition)

$$\hat{\mathbf{p}}^F_E = -\frac{(n^F)^2\gamma^{FR}}{k^F}\mathbf{w}_F$$

where: γ^{FR} : specific weight of the pore fluid
k^F : Darcy permeability

- **Solid consituent** (elasto-viscoplastic, small deformations)

  * decomposition of the strain tensor:

$$\boldsymbol{\varepsilon}_S = \boldsymbol{\varepsilon}_{Se} + \boldsymbol{\varepsilon}_{Sp}$$

  * solid extra stress (generalised Hookean law):

$$\mathbf{T}^S_E = 2\mu^S \boldsymbol{\varepsilon}_{Se} + \lambda^S(\boldsymbol{\varepsilon}_{Se}\cdot\mathbf{I})\mathbf{I}$$

where: μ^S, λ^S : Lamé constants

  * elastic strain tensor (small strains)

$$\boldsymbol{\varepsilon}_{Se} = \tfrac{1}{2}(\mathrm{grad}\,\mathbf{u}_S + \mathrm{grad}^T\mathbf{u}_S) - \boldsymbol{\varepsilon}_{Sp}$$

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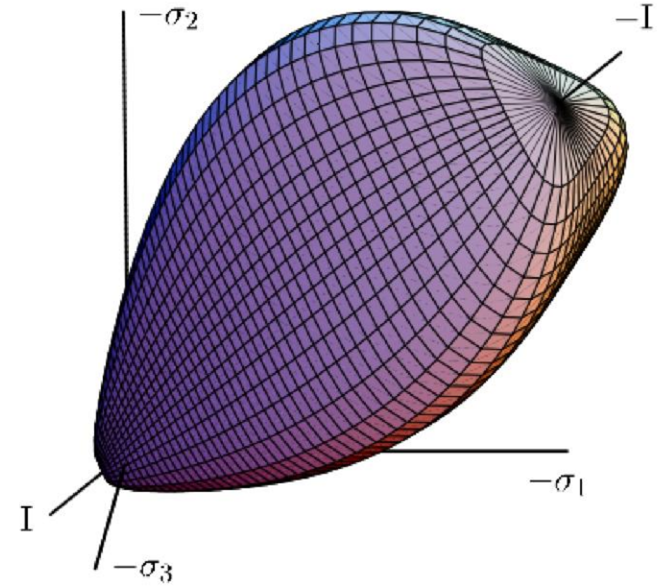
∗ single-surface yield criterion [*Ehlers* 1993]:

$$F = \Phi^{1/2} + \beta\,\mathrm{I} + \epsilon\,\mathrm{I}^2 - \kappa$$

$$\Phi = \mathrm{II}^D(1+\gamma\vartheta)^m + \tfrac{1}{2}\alpha\,\mathrm{I}^2 + \delta^2\mathrm{I}^4$$

$$\vartheta = \mathrm{III}^D/(\mathrm{II}^D)^{3/2}$$

principal invariants: $\mathrm{I}, \mathrm{II}^D, \mathrm{III}^D$ of $\mathbf{T}_E^S$



∗ parameters of the yield surface:

hydrostatic plane: $\boldsymbol{\mathcal{S}}_h = (\delta, \epsilon, \beta, \alpha, \kappa)^T$
deviatoric plane: $\boldsymbol{\mathcal{S}}_d = (\gamma, m)^T$

∗ plastic potential [*Ehlers & Mahnkopf* 1999]:

$$G = \sqrt{\psi_1\mathrm{II}^D + \tfrac{1}{2}\alpha\,\mathrm{I}^2 + \delta^2\mathrm{I}^4} + \psi_2\beta\,\mathrm{I} + \epsilon\,\mathrm{I}^2$$

∗ flow rule and plastic multiplier:

$$(\boldsymbol{\varepsilon}_{Sp})'_S = \Lambda\frac{\partial G}{\partial \mathbf{T}_E^S}$$

visco-plasticity [*Perzyna* 1966]:

$$\Lambda = \frac{1}{\eta}\left\langle\frac{F}{\sigma_0}\right\rangle^r$$

Numerical Treatment

➢ Variational Approach based on the *Bubnov-Galerkin* Method

- **Strong formulations:**
  - ∗ solid displacement-velocity relation (order reduction):

    $$(\mathbf{u}_S)'_S = \mathbf{v}_S$$

  - ∗ total momentum balance:

    $$\rho^S(\mathbf{v}_S)'_S + \rho^F[(\mathbf{v}_F)'_S + (\mathrm{grad}\,\mathbf{v}_F)\mathbf{w}_F] = \mathrm{div}\,(\mathbf{T}^S_E - p\,\mathbf{I}) + (\rho^S + \rho^F)\mathbf{b}$$

  - ∗ fluid momentum balance:

    $$\rho^F(\mathbf{v}_F)'_S + \rho^F(\mathrm{grad}\,\mathbf{v}_F)\mathbf{w}_F = \mathrm{div}(-n^F p\,\mathbf{I}) + \rho^F\,\mathbf{b} - \frac{(n^F)^2\gamma^{FR}}{k^F}\mathbf{w}_F + p\,\mathrm{grad}\,n^F$$

  - ∗ total volume balance:

    $$0 = \mathrm{div}(\mathbf{v}_S + \underbrace{\frac{k^F}{\gamma^{FR}}\{\rho^{FR}[\mathbf{b} - (\mathbf{v}_F)'_S - (\mathrm{grad}\,\mathbf{v}_F)\mathbf{w}_F] - \mathrm{grad}\,p\}}_{n^F\mathbf{w}_F})$$

- **Primary variables:**

  $$\mathbf{u}_S, \quad \mathbf{v}_S, \quad \mathbf{v}_F, \quad p$$

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- **Weak formulations**
  - $*$ displacement-velocity relation (test function $\delta\mathbf{u}_S$):

$$\mathcal{G}_{\mathbf{v}_S} = \int_\Omega \delta\mathbf{u}_S \cdot [(\mathbf{u}_S)'_S - \mathbf{v}_S]\mathrm{d}v = 0$$

  - $*$ total momentum balance (test function $\delta\mathbf{u}_S$):

$$\mathcal{G}_{\mathbf{u}_S} = \int_\Omega \delta\mathbf{u}_S \cdot \{\rho^S[(\mathbf{v}_S)'_S - \mathbf{b}] + \rho^F[(\mathbf{v}_F)'_S + (\mathrm{grad}\,\mathbf{v}_F)\mathbf{w}_F - \mathbf{b}]\}\mathrm{d}v +$$

$$+ \int_\Omega \mathrm{grad}\,\delta\mathbf{u}_S \cdot (\mathbf{T}^S_E - p\,\mathbf{I})\mathrm{d}v + \int_{\Gamma^S_\mathbf{t}} \delta\mathbf{u}_S \cdot \underbrace{(\mathbf{T}^S_E - p\,\mathbf{I})\,\mathbf{n}}_{\bar{\mathbf{t}}}\,\mathrm{d}a = 0$$

  - $*$ fluid momentum balance (test function $\delta\mathbf{v}_F$):

$$\mathcal{G}_{\mathbf{v}^F} = \int_\Omega \delta\mathbf{v}_F \cdot [\rho^F(\mathbf{v}_F)'_S + (\mathrm{grad}\,\mathbf{v}_F)\mathbf{w}_F - \mathbf{b}]\mathrm{d}v + \int_\Omega \delta\mathbf{v}_F \cdot [\frac{(n^F)^2\gamma^{FR}}{k^F}\mathbf{w}_F - p\,\mathrm{grad}\,n^F]\mathrm{d}v +$$

$$+ \int_\Omega \mathrm{div}\,\delta\mathbf{v}_F(-n^F p)\mathrm{d}v + \int_{\Gamma^F_\mathbf{t}} \delta\mathbf{v}_F \cdot \underbrace{(-\,n^F p\,\mathbf{I})\,\mathbf{n}}_{\bar{\mathbf{t}}^F}\,\mathrm{d}a = 0$$

  - $*$ total volume balance (test function δp)

$$\mathcal{G}_p = \int_\Omega \delta p\,\mathrm{div}\,\mathbf{v}_S\mathrm{d}v - \int_\Omega \mathrm{grad}\,\delta p \cdot \frac{k^F}{\gamma^{FR}}\{\mathrm{grad}\,p - \rho^{FR}[\mathbf{b} - (\mathbf{v}_F)'_S - (\mathrm{grad}\,\mathbf{v}_F)\mathbf{w}_F]\}\mathrm{d}v +$$

$$+ \int_{\Gamma_v} \delta p\,\underbrace{n^F\mathbf{w}_F \cdot \mathbf{n}}_{\bar{v}}\,\mathrm{d}a = 0$$

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Spatial Discretisation

- **Approximation of the unknown fields within small but finite subdomains**

  ∗ trial function: $\boldsymbol{u}(\mathbf{x},t) \approx \boldsymbol{u}^h(\mathbf{x},t) = \overline{\boldsymbol{u}}^h(\mathbf{x},t) + \sum_{i=1}^N \boldsymbol{\Phi}^i(\mathbf{x})\,\hat{\boldsymbol{u}}^i(t)$

  ∗ test function: $\delta\boldsymbol{u}(\mathbf{x}) \approx \delta\boldsymbol{u}^h(\mathbf{x}) = \sum_{i=1}^N \boldsymbol{\Phi}^i(\mathbf{x})\,\delta\hat{\boldsymbol{u}}^i$

  with: $\boldsymbol{\Phi}^i(\mathbf{x})$: matrix containing the shape functions
  $\hat{\boldsymbol{u}}^i(t)/\delta\hat{\boldsymbol{u}}^i$: vector containing the nodal degrees of freedom

- **Approximation of the integral of the weak forms**

$$\int_{\Omega^e} \boldsymbol{g}(\mathbf{x},t)\mathrm{d}v \approx \sum_{p=1}^{\bar{P}} w_p\, \boldsymbol{g}(\mathbf{x}_p,t)$$

  with: w_p : weight of an integration point
  $\bar{P}$: number of integration points of an element

- **Semi-discrete initial-boundary-value problem**

$$\boldsymbol{F}^h(t,\boldsymbol{u},\boldsymbol{u}',\boldsymbol{q},\boldsymbol{q}') = \begin{bmatrix} \boldsymbol{\mathcal{G}}^h(t,\,\boldsymbol{u},\,\boldsymbol{u}',\,\boldsymbol{q}) \\ \boldsymbol{\mathcal{L}}^h(t,\,\boldsymbol{q},\,\boldsymbol{q}',\,\boldsymbol{u}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{M}\boldsymbol{u}' + \boldsymbol{K}(\boldsymbol{u},\boldsymbol{q}) - \boldsymbol{f} \\ \boldsymbol{A}\boldsymbol{q}' + \boldsymbol{G}(\boldsymbol{q},\boldsymbol{u}) \end{bmatrix} \overset{!}{=} \mathbf{0}$$

with: $\boldsymbol{u} = [(\hat{\mathbf{u}}_S^1, \hat{\mathbf{v}}_S^1, \hat{\mathbf{v}}_F^1, \hat{\mathrm{p}}^1), \cdots, (\hat{\mathbf{u}}_S^N, \hat{\mathbf{v}}_S^N, \hat{\mathbf{v}}_F^N, \hat{\mathrm{p}}^N)]^T$

$\boldsymbol{q} = [(\boldsymbol{\varepsilon}_{Sp}^1,\, \Lambda^1), \cdots, (\boldsymbol{\varepsilon}_{Sp}^P,\, \Lambda^P)]^T$

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➢ Solution Strategy

- Splitting the overall Differential-Algebraic System of Equations (DAE) into a local and a global system

global system

$$\boldsymbol{F}^h(t,\boldsymbol{u},\boldsymbol{u}',\boldsymbol{q},\boldsymbol{q}') = \begin{bmatrix} \boldsymbol{\mathcal{G}}^h(t,\boldsymbol{u},\boldsymbol{u}',\boldsymbol{q}) \\ \boldsymbol{\mathcal{L}}^h(t,\boldsymbol{u},\boldsymbol{q},\boldsymbol{q}') \end{bmatrix} = \begin{bmatrix} \boldsymbol{M}\boldsymbol{u}' + \boldsymbol{K}(\boldsymbol{u},\boldsymbol{q}) - \boldsymbol{f} \\ \boldsymbol{A}\,\boldsymbol{q}' + \boldsymbol{G}(\boldsymbol{q},\boldsymbol{u}) \end{bmatrix} \overset{!}{=} \mathbf{0}$$

local system

- Structure of the global system of equations

$$\boldsymbol{\mathcal{G}}^h = \underbrace{\begin{bmatrix} \boldsymbol{I} & 0 & 0 & 0 \\ 0 & \boldsymbol{M}_{22} & \boldsymbol{M}_{23} & 0 \\ 0 & 0 & \boldsymbol{M}_{33} & 0 \\ 0 & 0 & \boldsymbol{M}_{43} & 0 \end{bmatrix}}_{\boldsymbol{M}} \underbrace{\begin{bmatrix} \dot{\mathbf{u}}_S \\ \dot{\mathbf{v}}_S \\ \dot{\mathbf{v}}_F \\ \dot{\mathbf{p}} \end{bmatrix}}_{\boldsymbol{u}'} + \underbrace{\begin{bmatrix} 0 & -\boldsymbol{I} & 0 & 0 \\ \boldsymbol{K}_{21} & \boldsymbol{K}_{22} & \boldsymbol{K}_{23} & \boldsymbol{K}_{24} \\ 0 & \boldsymbol{K}_{32} & \boldsymbol{K}_{33} & \boldsymbol{K}_{34} \\ 0 & \boldsymbol{K}_{42} & \boldsymbol{K}_{43} & \boldsymbol{K}_{44} \end{bmatrix}}_{\boldsymbol{K}} \underbrace{\begin{bmatrix} \mathbf{u}_S \\ \mathbf{v}_S \\ \mathbf{v}_F \\ \mathbf{p} \end{bmatrix}}_{\boldsymbol{u}} - \underbrace{\begin{bmatrix} 0 \\ \boldsymbol{b}_1 + \boldsymbol{f}_{S+F} \\ \boldsymbol{b}_2 + \boldsymbol{f}_F \\ \boldsymbol{b}_3 + \boldsymbol{f}_p \end{bmatrix}}_{\boldsymbol{f}} \overset{!}{=} \mathbf{0}$$

- Structure of the local system of equations (element level)

$$\boldsymbol{\mathcal{L}}^h = \boldsymbol{A}\,\boldsymbol{q}' + \boldsymbol{G}(\boldsymbol{q},\boldsymbol{u}) = \begin{bmatrix} (\boldsymbol{\varepsilon}_{Sp})' \\ 0 \end{bmatrix} - \begin{bmatrix} \Lambda \dfrac{\partial G}{\partial \mathbf{T}_E^S} \\ \dfrac{1}{\eta}\left\langle \dfrac{F(\mathbf{T}_E^S)}{\sigma_0} \right\rangle^r - \Lambda \end{bmatrix} \overset{!}{=} \mathbf{0}$$

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➢ Temporal Discretisation

- **Temporal discretisation of the local system of equations**

  ∗ backward Euler (implicit):

$$\mathcal{L}_n^h\,(t_n,\,\boldsymbol{q}_n,\,\boldsymbol{q}_n',\,\boldsymbol{u}_n) = \boldsymbol{A}\,\boldsymbol{q}_n' + \boldsymbol{G}(\boldsymbol{q}_n,\boldsymbol{u}_n) = \boldsymbol{0} \qquad \text{with:} \quad \boldsymbol{q}_n' \approx \frac{\boldsymbol{q}_n - \boldsymbol{q}_{n-1}}{\Delta t_n} = \frac{\Delta \boldsymbol{q}_n}{\Delta t_n}$$

$$\boldsymbol{q}_n \approx \boldsymbol{q}_{n-1} + \Delta \boldsymbol{q}_n$$

$$\Longrightarrow \quad \mathcal{L}_n^h\,\Big(t_n\,,\ \boldsymbol{q}_{n-1} + \Delta\boldsymbol{q}_n\,,\ \frac{1}{\Delta t_n}\Delta\boldsymbol{q}_n\,,\ \boldsymbol{u}_{n-1} + \Delta\boldsymbol{u}_n\Big) \stackrel{!}{=} \boldsymbol{0}$$

$\Longrightarrow$ solve for $\Delta\boldsymbol{q}_n$ while $\Delta\boldsymbol{u}_n$ is kept constant

$\Longrightarrow$ update: $\boldsymbol{q}_n = \boldsymbol{q}_{n-1} + \Delta\boldsymbol{q}_n$

- **Temporal discretisation of the global system of equations**

  ∗ backward Euler (implicit):

$$\mathcal{G}_n^h\,(t_n,\,\boldsymbol{u}_n,\,\boldsymbol{u}_n',\,\boldsymbol{q}_n) = \boldsymbol{M}\boldsymbol{u}_n' + \boldsymbol{K}\boldsymbol{u}_n - \boldsymbol{f}_n = \boldsymbol{0} \qquad \text{with:} \quad \boldsymbol{u}_n' \approx \frac{\boldsymbol{u}_n - \boldsymbol{u}_{n-1}}{\Delta t_n} = \frac{\Delta \boldsymbol{u}_n}{\Delta t_n}$$

$$\boldsymbol{u}_n \approx \boldsymbol{u}_{n-1} + \Delta \boldsymbol{u}_n$$

$$\Longrightarrow \quad \mathcal{G}_n^h\,\Big(t_n\,,\ \boldsymbol{u}_{n-1} + \Delta\boldsymbol{u}_n\,,\ \frac{1}{\Delta t_n}\Delta\boldsymbol{u}_n\,,\ \boldsymbol{q}_{n-1} + \Delta\boldsymbol{q}_n\Big) \stackrel{!}{=} \boldsymbol{0}$$

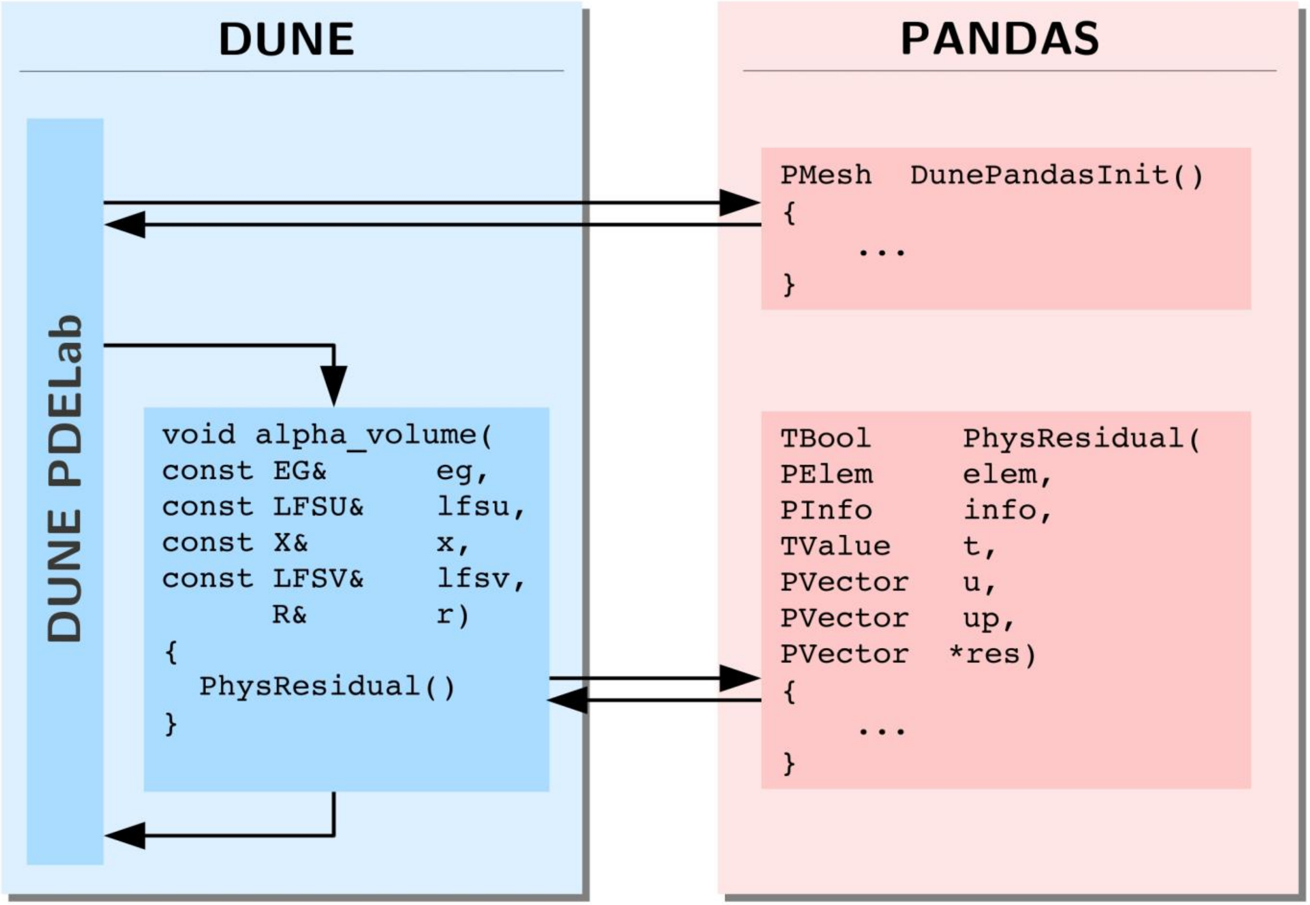
$\Longrightarrow$ solve for $\Delta\boldsymbol{u}_n$ while $\Delta\boldsymbol{q}_n$ is kept constant

$\Longrightarrow$ update: $\boldsymbol{u}_n = \boldsymbol{u}_{n-1} + \Delta\boldsymbol{u}_n$

Interface

➢ DUNE-PANDAS-Interface Workflow

∗ intersection of DUNE and PANDAS is the local operator of DUNE PDELab



⟹ functionality of `alpha_volume()` will be replaced by `PhysResidual()`

Data transfer

∗ data containers of `alpha_volume()` and `PhysResidual()`

`alpha_volume()`		`PhysResidual()`	
`eg`:	pointer on grid entity	`elem`:	pointer on element: nodal coordinates, trial- and test functions, history variables (q_{n-1}), secondary variables
`lfsu`:	pointer on trial functions of local function space		
`lfsv`:	pointer on test functions of local function space		
`t`:	value of current time	`t`:	value of current time
`x`:	pointer on... 1) current solution (local operator) 2) time derivative of current sol. (time local operator)	`u`:	pointer on cur. solution (u_n)
		`up`:	pointer on time derivative of current solution (u'_n)
`r`:	pointer on internal response (residual)	`res`:	pointer on internal response (residual)

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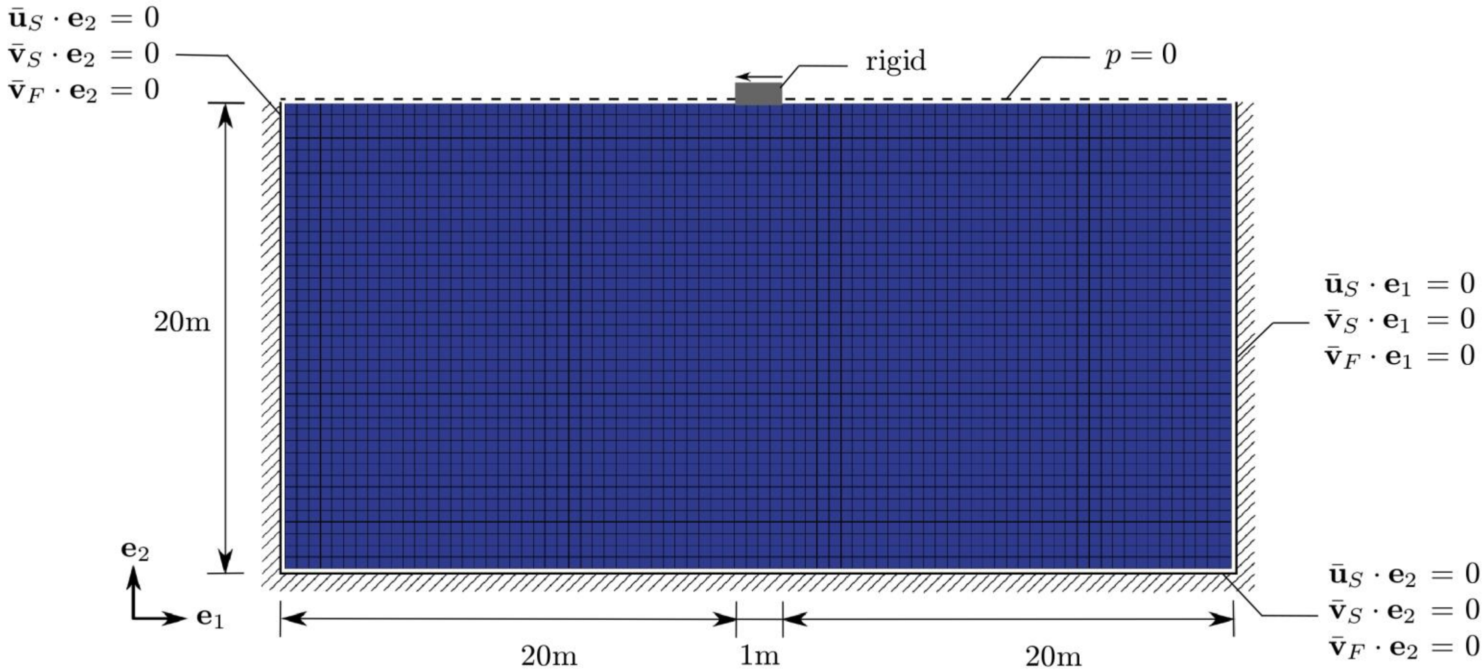
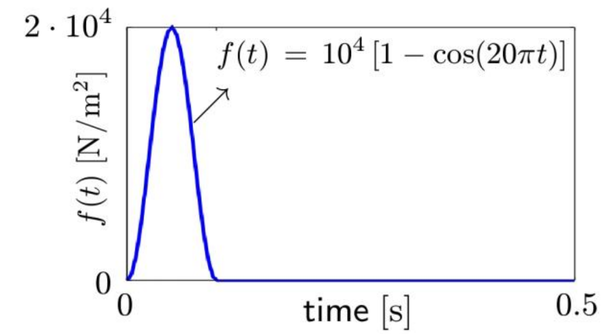
➢ Two-dimensional Elastic Shear-Wave Propagation



- **Parameters:**

Parameter	Abb.	Value	Unit
1st Lamé constant	μ^S	$5.583 \cdot 10^6$	$\mathrm{N/m^2}$
2nd Lamé constant	λ^S	$8.375 \cdot 10^6$	$\mathrm{N/m^2}$
solid effective density	ρ^{SR}	2720	$\mathrm{kg/m^3}$
fluid effective density	ρ^{FR}	1000	$\mathrm{kg/m^3}$
initial volume fraction	n^S_{0S}	0.67	
Darcy permeability	k^F	10^{-2}	$\mathrm{m/s}$

- **Load:**



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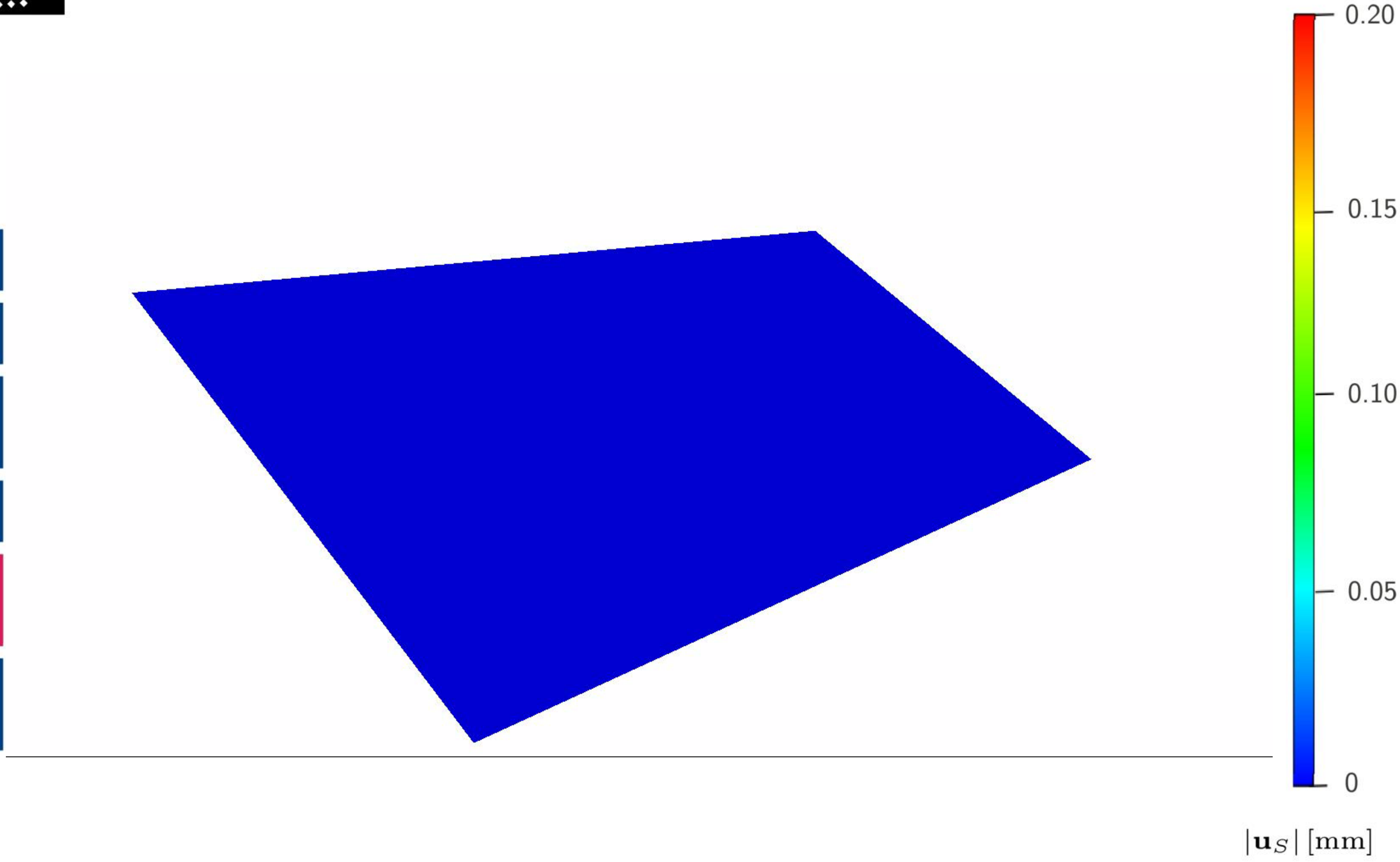
➢ Result



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Summary

- Theory of porous media
- Numerical treatment (discretisation in space and time)
- DUNE-PANDAS Interface (procedure overview)
- Numerical example (wave propagation)

Outlook

- Complete data transfer between DUNE and PANDAS (history variables, etc.)
- Parallel execution

Thank you for your attention!