Using DUNE and PDELab for the diploma thesis as a student

DUNE User Meeting

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Outline

- **Prelude**
- Two-phase flows in porous media
- Numerical results

Conditions

Use of DUNE and PDELab with regard to

- Diploma thesis in computer science (*Diplomarbeit in Informatik*)
- Solve PDE from two-phase flows in porous media
- Use of Discontinuous Galerkin method (DG)
- Six months time frame
- No direct DUNE supervision

Existing works

- Parabolic SIPG examples in pressure-saturation-form (Y. Epshteyn 2007)
- Elliptic SIPG examples in saturation-global-pressure form with discontinuous capillary pressures (A. Ern et al. 2010)

Scholastic prelude

What I did so far

- Eight semesters computer science
- Six semesters mathematics
- No experience with instationary PDEs

What I had so far

- Sound C++ knowledge
- *Studienarbeit*: Elliptic Darcy problem with PDELab and DG
- Finite volume example code for two-phase Darcy flows

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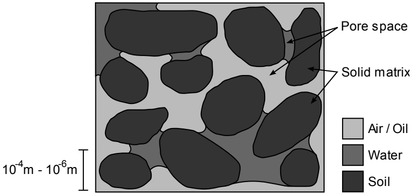
- Prelude
- **Two-phase flows in porous media**
- Numerical results

Porous media

- Connected pores filled with non-miscible fluids

$$S_\alpha + S_\beta = 1 \tag{1}$$

- Averaged quantities: porosity Φ, saturation S, fluid velocity u
- Absolute permeability K may have huge jumps



Relative permeability and capillary pressures

Relative permeability $k_{rw}(S_w)$ / $k_{rn}(S_w)$

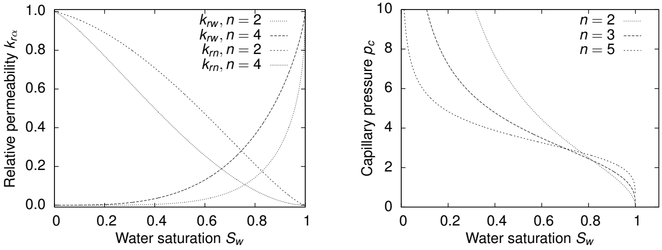
- Depending on saturation
- Non-linear

Capillary pressures $p_c(S_w)$

- Between wetting and non-wetting phase e. g. water / oil, water / air

$$p_c = p_n - p_w \tag{2}$$

- Singularity for $S_w \to 0$



Equations for two-phase Darcy flows

$$\frac{\partial}{\partial t}\left(\Phi \rho_\alpha S_\alpha\right) - \nabla \cdot \left(\rho_\alpha \lambda_\alpha(S_\alpha) K \left(\nabla p_\alpha - \rho_\alpha g\right)\right) = q_\alpha \quad \text{for } \alpha \in \{w, n\} \tag{3}$$

Substituting $S_w + S_n = 1$ and $p_c = p_n - p_w$:

$$\frac{\partial}{\partial t}\left(\Phi \rho_w \left(1 - S_n\right)\right) - \nabla \cdot \left(\rho_w \lambda_w(1 - S_n) K \left(\nabla p_w - \rho_w g\right)\right) = q_w \tag{4a}$$

$$\frac{\partial}{\partial t}\left(\Phi \rho_n S_n\right) - \nabla \cdot \left(\rho_n \lambda_n(S_n) K \left(\nabla p_w + \nabla p_c(1 - S_n) - \rho_n g\right)\right) = q_n \tag{4b}$$

- Pressure-saturation-form
- Independent variables p_w and S_n
- Other forms are possible

Richards equation

- Simplified model for water movement in unsaturated soil
- Air pressure considered constant and defined as $p_n := 0$
- Saturation $S\left(p_w\right) := p_c^{-1}\left(-p_w\right)$

$$\frac{\partial}{\partial t} S(p_w) - \nabla \cdot \big(K(p_w) \left(\nabla p_w - z \right) \big) = q_w \tag{5}$$

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- Prelude
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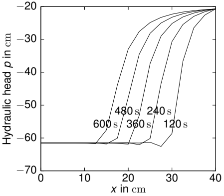
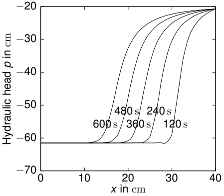
- **Numerical results**

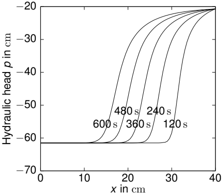
Richards: Haverkamp's example



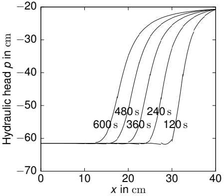
SIPG 1



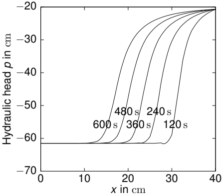
SIPG 2



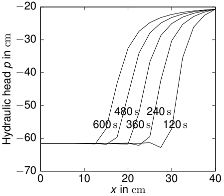
SIPG 3



OBB 2

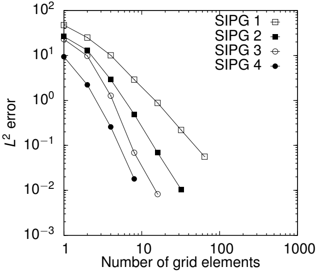


NIPG 2

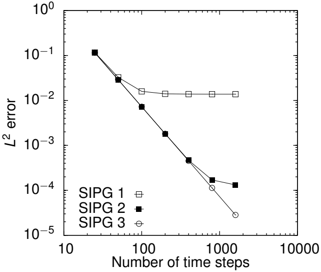


Finite element Q1

Richards: Example with analytic solution

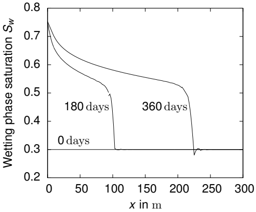


L^2 error $h \to 0$

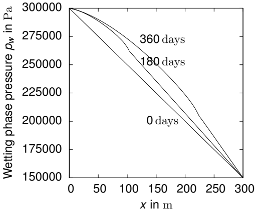


L^2 error $\Delta t \to 0$

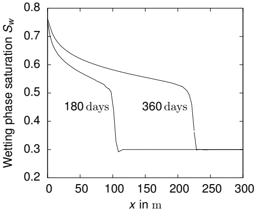
Two-phase: Buckley Leverett example



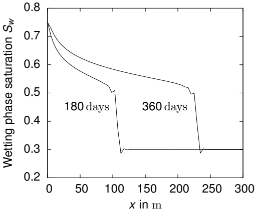
NIPG 2, saturation



NIPG 2, pressure

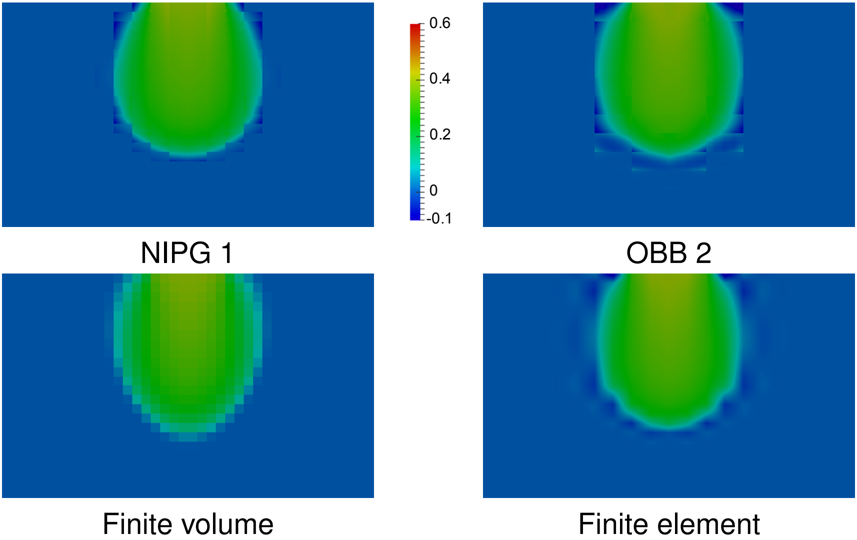


OBB 5

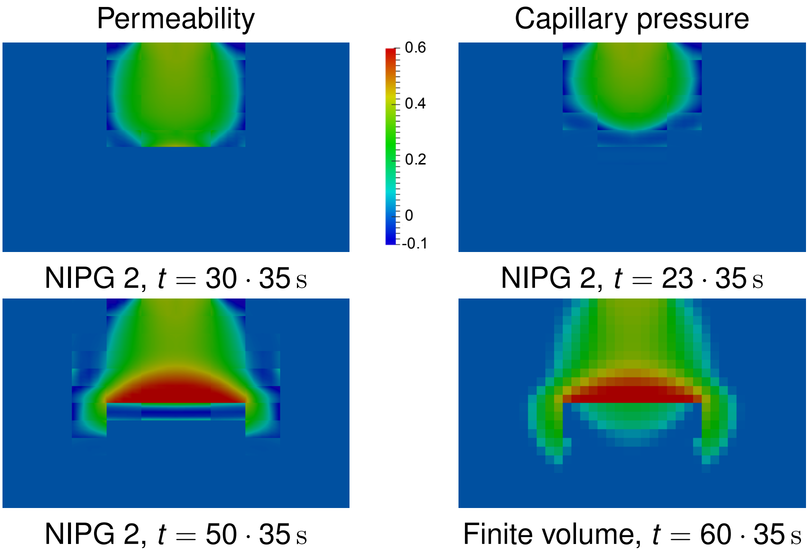


Finite element Q1

Two-phase: 2D infiltration example



With sand lense



Conclusions

General

- Two-phase flows in porous media can be done in six months
- Starting with basic programs more in-deep research possible
- Either broad C++ skills or experience with DUNE are necessary

Personal

- Despite some open-end questions I consider it a success
- Precious experience and qualification
- Shallow water equations programmed within three weeks

Thanks for your attention

Questions?