Upscaling of Porous Media Flow with DuMu^x and dune-multidomaingrid

Bernd Flemisch, Markus Wolff

Dune User Meeting, 6.-8.10.10, Stuttgart

Outline

- DuMu^x
- Tensorial Permeabilities
- Upscaling Concept
- Implementation Concept
- Summary

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DuMu^x

Dune for **Mu**lti-{phase, component, scale, physics, …} Flow in Porous Media

Used / developed by:

- Katherina Baber
- Melanie Darcis
- Karin Erbertseder
- Benjamin Faigle
- Bernd Flemisch
- Andreas Lauser
- Klaus Mosthaf
- Philipp Nuske
- Sergey Oladyshkin
- Nicolas Schwenck
- Alex Tatomir
- Lena Walter
- Markus Wolff

- Yufei Cao
- Marc Schlienger

- Leopold Stadler

LH2

NMH

DuMu^x

dumux.org

- **1/2007**: started
- **~4300**: svn revision
- **~50,000** lines (stable)
- **1.7.09**: release 1.0
- soon: next release

Coupled Fully Implicit Models

- **Implicit** Euler time discretization
- Vertex centered FV (**box**) space discretization
- **Newton** method
- **Models**:
  - One phase: 1p, 1p2c
  - Two phases: 2p, 2pni, 2p2c, 2p2cni, (2p3c, 2pia, 2p2cia)
  - (MpNc)
  - (Double continuum, MINC, DFN)
  - (Richards equation)

Decoupled Semi-implicit Models

- **Standard** and **phase pressure** fractional flow formulation
- Twophase(-**twocomponent-nonisothermal**), (3p3c in progress)
- (**Multiscale** approaches for pressure and transport equations)
- (**Multiphysics** approaches like 2p2c – 1p2c coupling)
- **Transport** equation:
  - Cell centered FV, implicit and explicit
  - Vertex centered FV, implicit
  - (Characteristic method)
  - (Operator splitting framework)
- **Pressure** equation:
  - Cell centered FV (with MPFA (2D))
  - Mimetic FD (2D)
  - (P1 FE with post-processed conservative fluxes)

(Other Models and Capabilities)

- Parker-Lenhard type **hysteresis**
- **Brinkman** equation: coupling (single-phase) free flow with porous media flow
- **Coupling of free flow and porous media flow**:
  - Use of dune-multidomain
  - 2c free flow and 2p2c porous media flow
- **Matrix deformation**:
  - linear elasticity model implemented
  - Coupling to flow via dune-multidomain
- **Fractured media:**
  - Nonmatching (d-1) and d-dimensional grids
  - X-FEM approach using dune-multidomain

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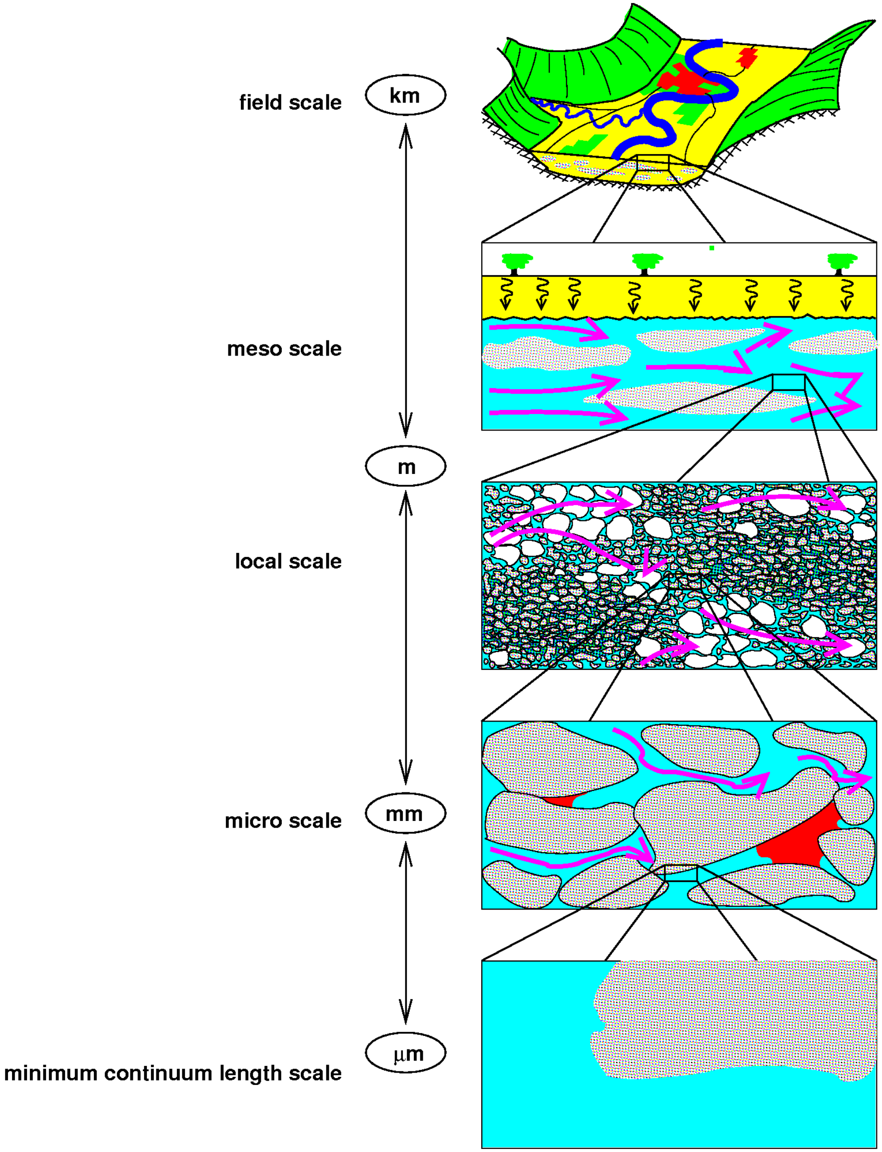
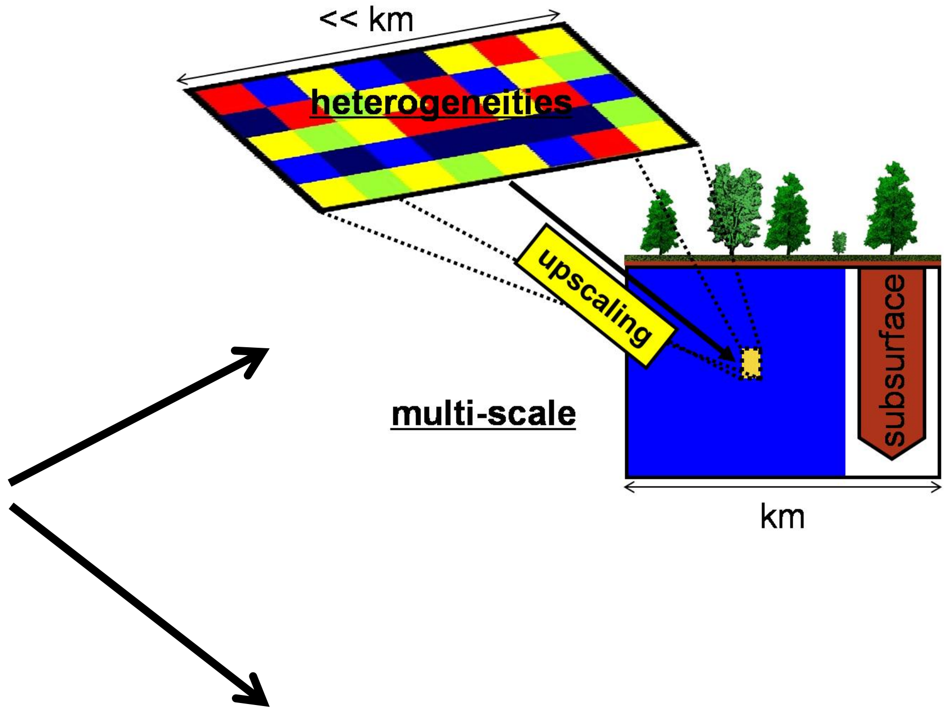
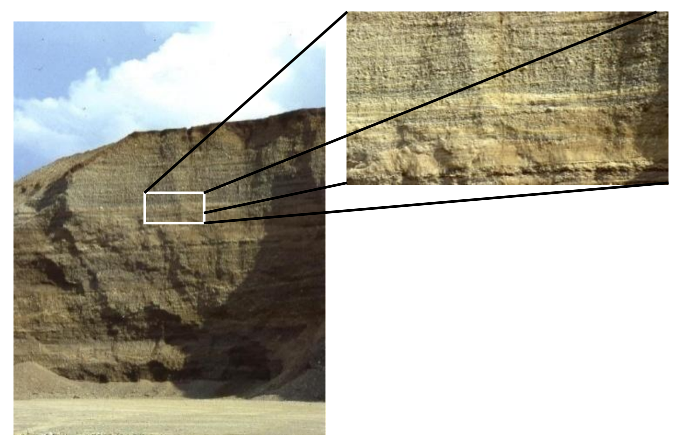
Scales

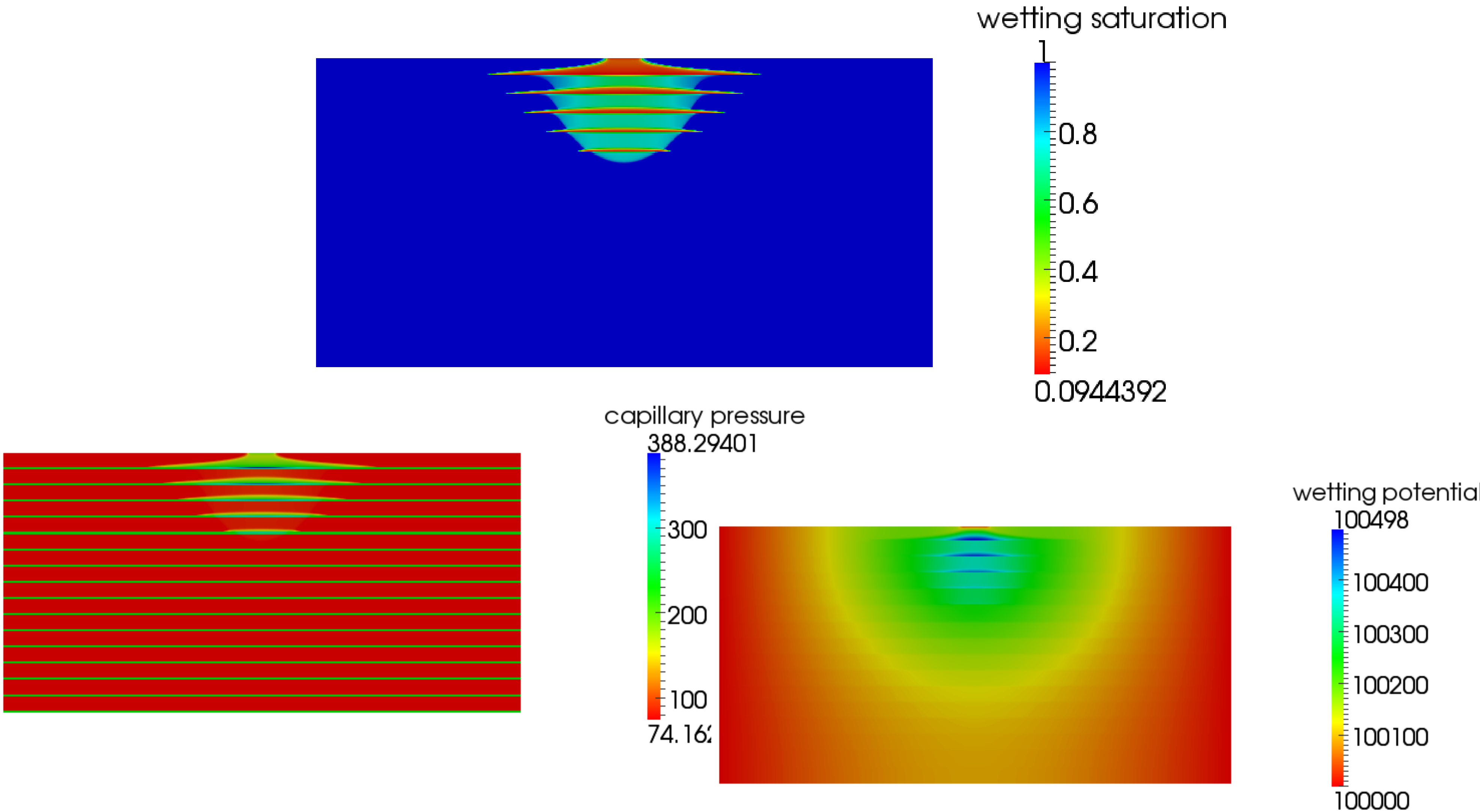


(Helmig)





Necessity of coarse-scale full tensor coefficients



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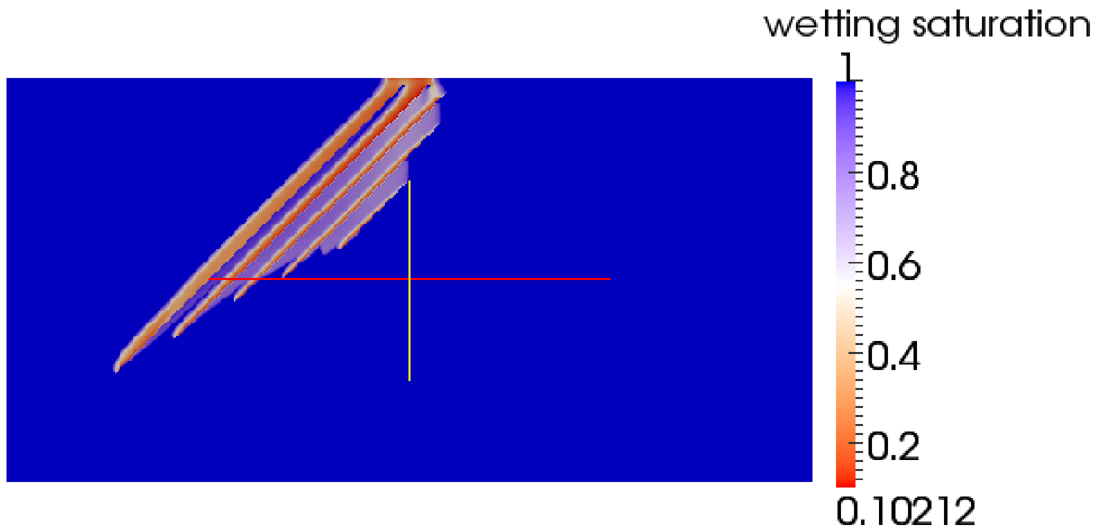

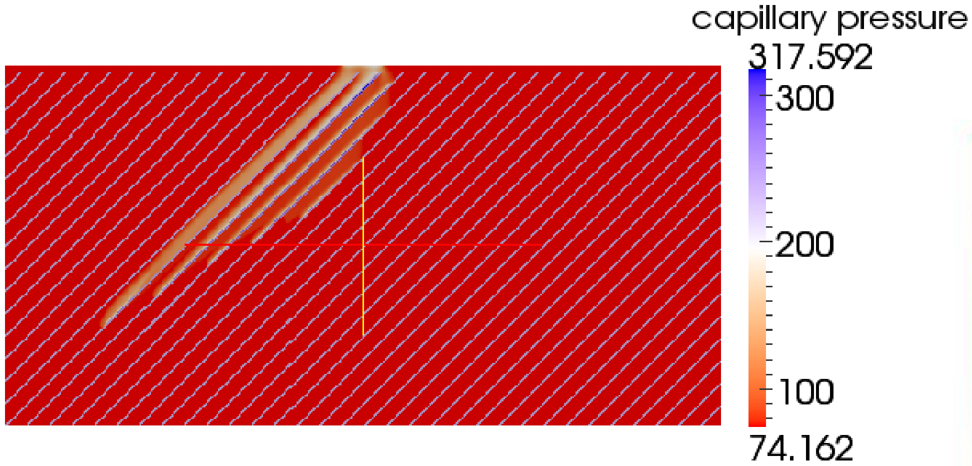

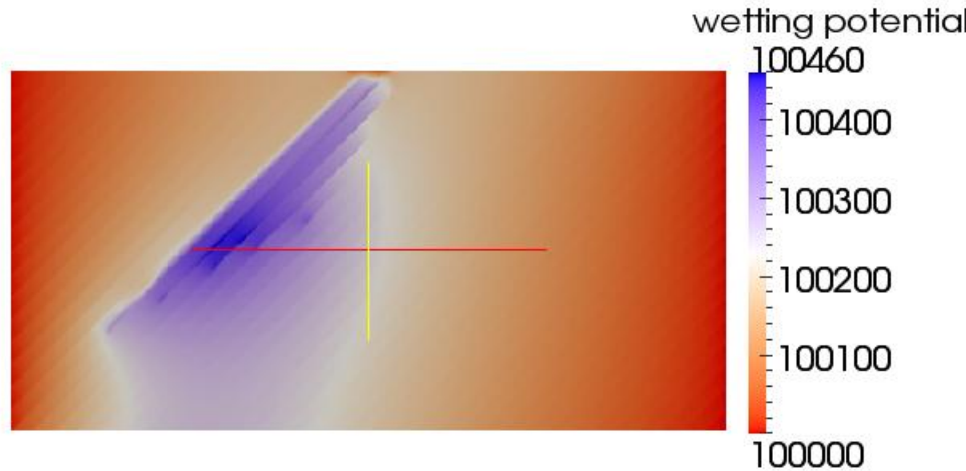

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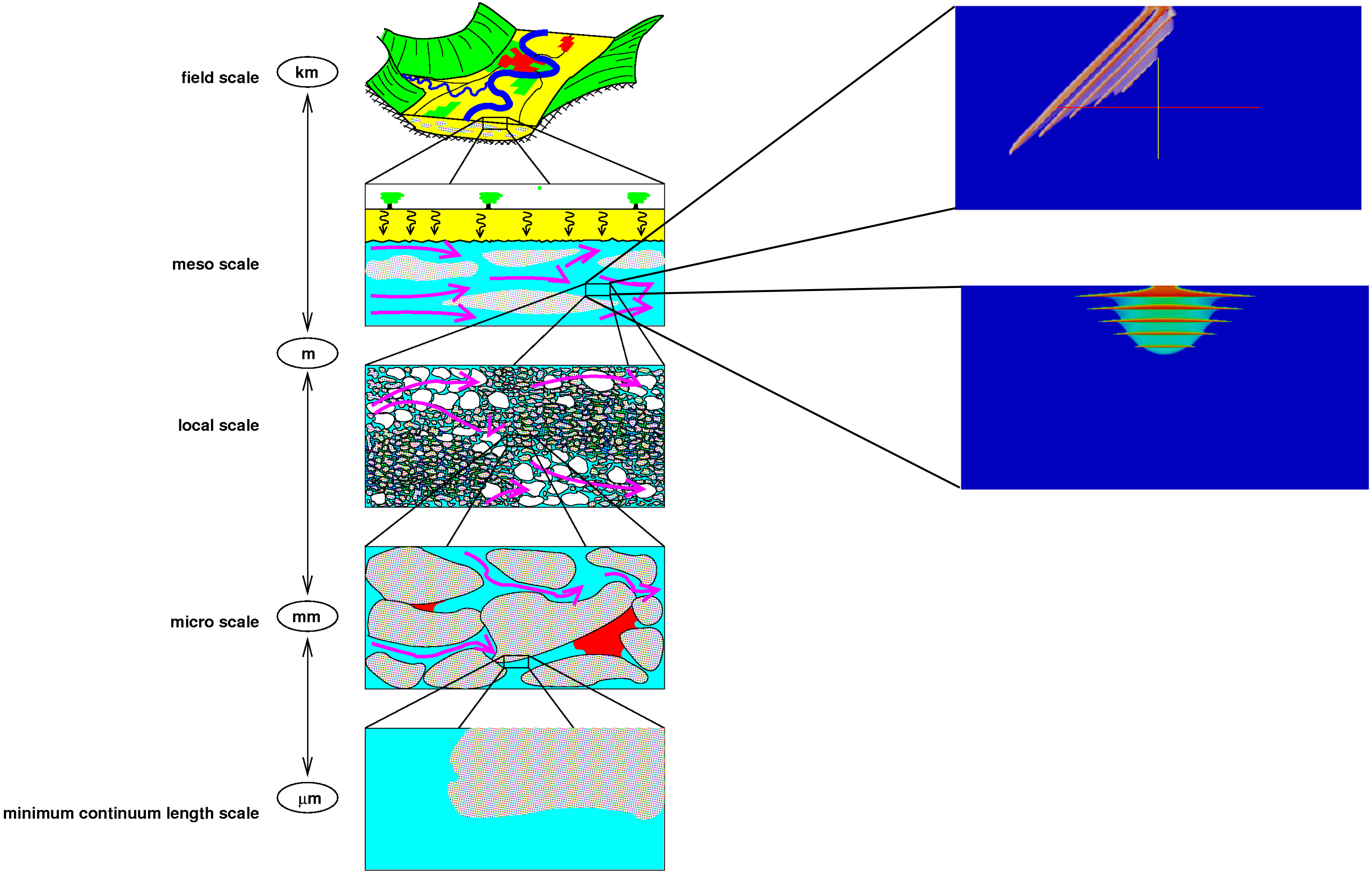

(Helmig)

Upscaled Equations

- Averaged equations

$$\frac{\partial(\phi^* S^*_{\mathrm{w}})}{\partial t} + \nabla \cdot \langle \mathbf{v}_{\mathrm{w}} \rangle = 0 \qquad \langle \mathbf{v}_{\mathrm{w}} \rangle = -\frac{\mathbf{K}^* \mathbf{K}^*_{r\mathrm{w}}}{\mu_{\mathrm{w}}} \cdot \left[\nabla \langle p_{\mathrm{w}} \rangle_{\mathrm{w}} - \varrho_{\mathrm{w}} g \nabla z\right]$$

$$\frac{\partial(\phi^* S^*_{\mathrm{n}})}{\partial t} + \nabla \cdot \langle \mathbf{v}_{\mathrm{n}} \rangle = 0 \qquad \langle \mathbf{v}_{\mathrm{n}} \rangle = -\frac{\mathbf{K}^* \mathbf{K}^*_{r\mathrm{n}}}{\mu_{\mathrm{n}}} \cdot \left[\nabla \langle p_{\mathrm{n}} \rangle_{\mathrm{n}} - \varrho_{\mathrm{n}} g \nabla z\right]$$

$$S^*_{\mathrm{n}} = \frac{V_{\mathrm{n}_{\mathrm{mobile}}}}{V_{\mathrm{pores}} - V_{\mathrm{immobile}}} \qquad S^*_{\mathrm{w}} = \frac{V_{\mathrm{w}_{\mathrm{mobile}}}}{V_{\mathrm{pores}} - V_{\mathrm{immobile}}}$$

$$p^*_{\mathrm{c}} = \langle p_{\mathrm{n}} \rangle_{\mathrm{n}} - \langle p_{\mathrm{w}} \rangle_{\mathrm{w}} \qquad S^*_{\mathrm{w}} + S^*_{\mathrm{n}} = 1 \qquad \langle \Psi_\alpha \rangle_\alpha = \frac{1}{V_\alpha} \int_{V_\alpha} \Psi_\alpha dV$$

Choice of Discretization

- Discretization must be able to treat the tensor coefficients!

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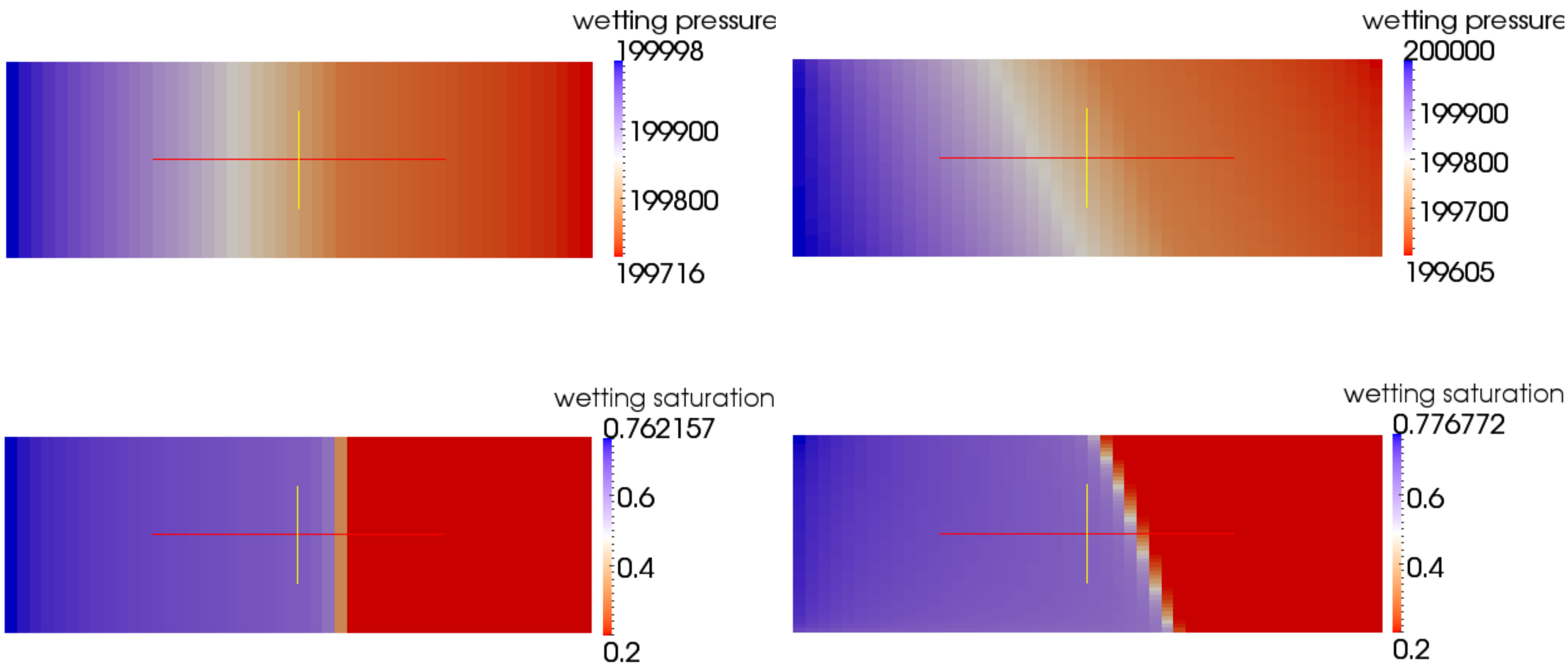

Choice of Discretization

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  - Two-Point Flux Approximation (TPFA) NOT sufficient!

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- Discretization must be able to treat the tensor coefficients!
  - Two-Point Flux Approximation (TPFA) NOT sufficient!

    ➔ Standard finite volume methods cannot be used
  - Alternatives:
    - Mixed finite elements
    - Mimetic finite differences
    - **Multi-Point-Flux-Approximation (MPFA)**

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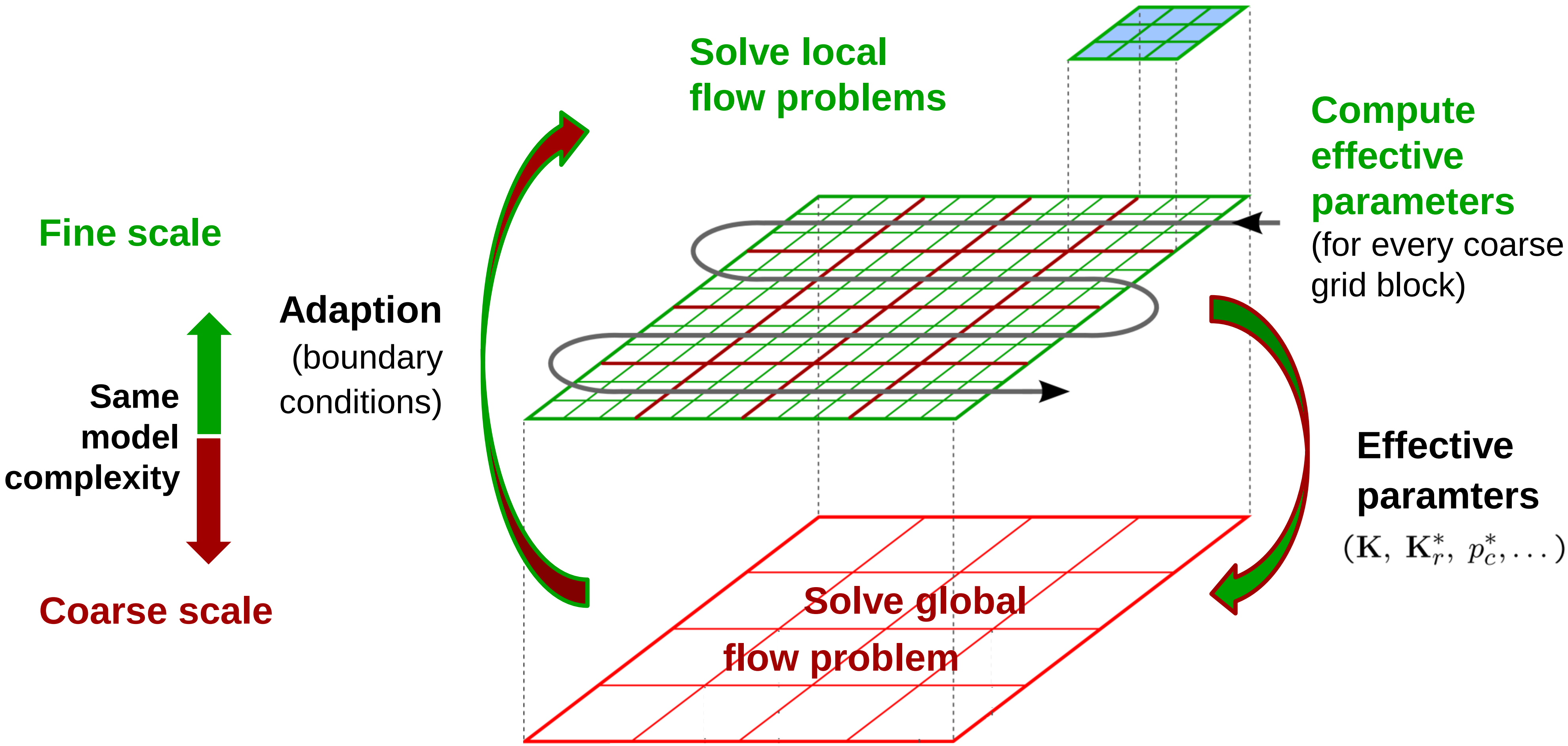
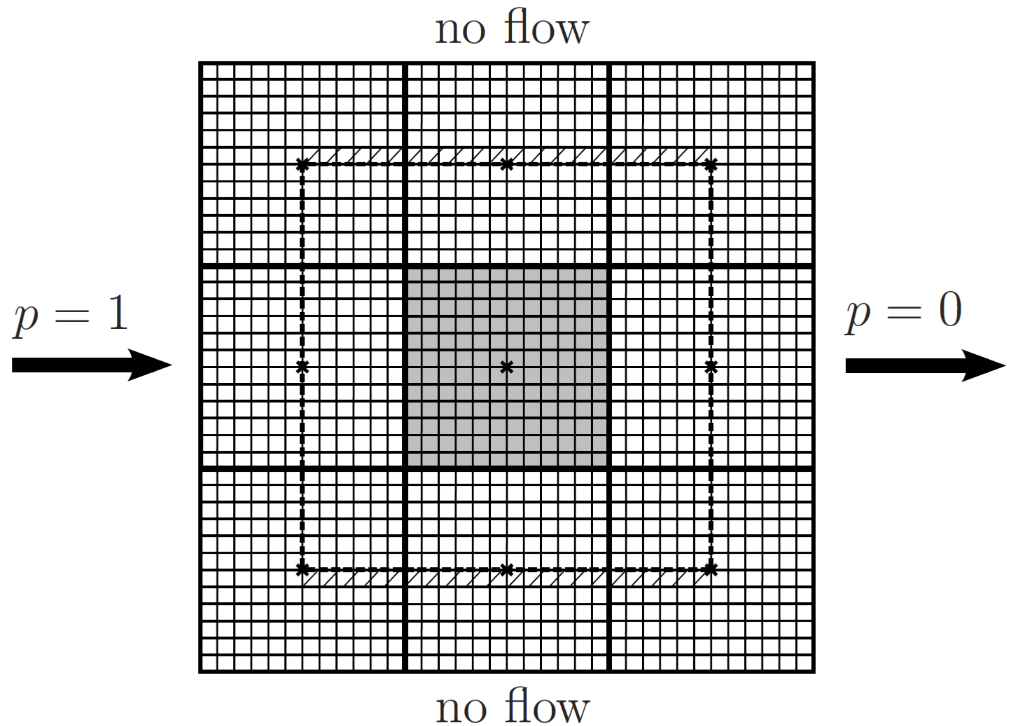
Upscaling Concept



Intrinsic permeability upscaling

- X. H. Wen, L. J. Durlofsky, 2003

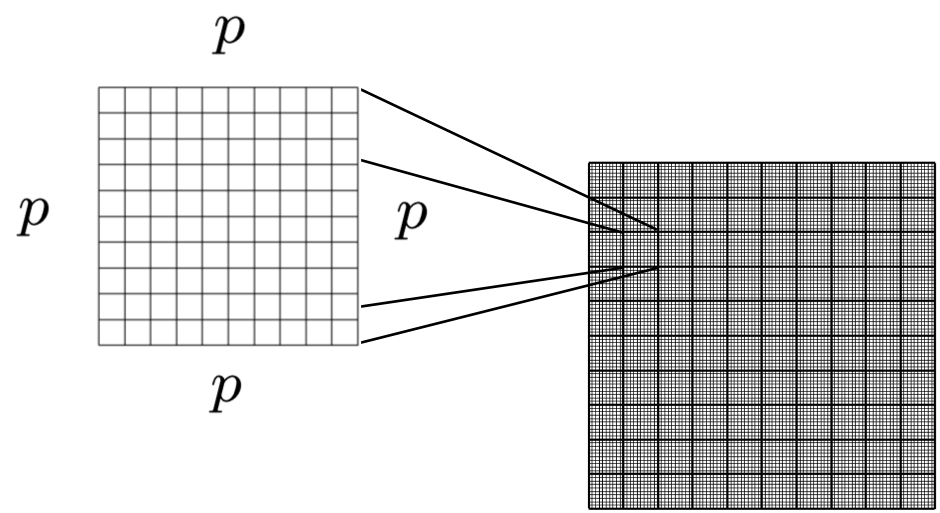


$$\begin{pmatrix} \left\langle \frac{\partial p}{\partial x} \right\rangle^x & \left\langle \frac{\partial p}{\partial y} \right\rangle^x & 0 & 0 \\ 0 & 0 & \left\langle \frac{\partial p}{\partial x} \right\rangle^x & \left\langle \frac{\partial p}{\partial y} \right\rangle^x \\ \left\langle \frac{\partial p}{\partial x} \right\rangle^y & \left\langle \frac{\partial p}{\partial y} \right\rangle^y & 0 & 0 \\ 0 & 0 & \left\langle \frac{\partial p}{\partial x} \right\rangle^y & \left\langle \frac{\partial p}{\partial y} \right\rangle^y \end{pmatrix} \begin{pmatrix} K^*_{xx} \\ K^*_{xy} \\ K^*_{yx} \\ K^*_{yy} \end{pmatrix} = - \begin{pmatrix} \langle v_x \rangle^x \\ \langle v_y \rangle^x \\ \langle v_x \rangle^y \\ \langle v_y \rangle^y \end{pmatrix}$$

$$K^*_{xy} = K^*_{yx} = (K^*_{xy} + K^*_{yx})/2$$

Relative permeability upscaling

- Use pressure field of global calculations for boundary conditions
- Calculate velocity field and solve system of eq.



$$\begin{pmatrix} \Psi^x_{\alpha,x} & \Psi^x_{\alpha,y} & 0 & 0 \\ 0 & 0 & \Psi^x_{\alpha,x} & \Psi^x_{\alpha,y} \\ \Psi^y_{\alpha,x} & \Psi^y_{\alpha,y} & 0 & 0 \\ 0 & 0 & \Psi^y_{\alpha,x} & \Psi^y_{\alpha,y} \end{pmatrix} \begin{pmatrix} K^*_{\mathrm{tot}_{xx\alpha}} \\ K^*_{\mathrm{tot}_{xy\alpha}} \\ K^*_{\mathrm{tot}_{yx\alpha}} \\ K^*_{\mathrm{tot}_{yy\alpha}} \end{pmatrix} = - \begin{pmatrix} \langle v_{\alpha x} \rangle^x_\alpha \\ \left\langle v_{\alpha y} \right\rangle^x_\alpha \\ \langle v_{\alpha x} \rangle^y_\alpha \\ \left\langle v_{\alpha y} \right\rangle^y_\alpha \end{pmatrix}$$

Relative permeability upscaling

$$\begin{pmatrix} \Psi^x_{\alpha,x} & \Psi^x_{\alpha,y} & 0 & 0 \\ 0 & 0 & \Psi^x_{\alpha,x} & \Psi^x_{\alpha,y} \\ \Psi^y_{\alpha,x} & \Psi^y_{\alpha,y} & 0 & 0 \\ 0 & 0 & \Psi^y_{\alpha,x} & \Psi^y_{\alpha,y} \end{pmatrix} \begin{pmatrix} K^*_{\mathrm{tot}_{xx\alpha}} \\ K^*_{\mathrm{tot}_{xy\alpha}} \\ K^*_{\mathrm{tot}_{yx\alpha}} \\ K^*_{\mathrm{tot}_{yy\alpha}} \end{pmatrix} = - \begin{pmatrix} \langle v_{\alpha x} \rangle^x_\alpha \\ \langle v_{\alpha y} \rangle^x_\alpha \\ \langle v_{\alpha x} \rangle^y_\alpha \\ \langle v_{\alpha y} \rangle^y_\alpha \end{pmatrix}$$

$$\Psi^x_{\alpha,x} = \frac{1}{\mu_\alpha} \left\langle \frac{\partial p_\alpha}{\partial x} \right\rangle^x_\alpha + \rho_\alpha g \nabla z \qquad \Psi^x_{\alpha,y} = \frac{1}{\mu_\alpha} \left\langle \frac{\partial p_\alpha}{\partial y} \right\rangle^x_\alpha + \rho_\alpha g \nabla z$$

$$\Psi^y_{\alpha,x} = \frac{1}{\mu_\alpha} \left\langle \frac{\partial p_\alpha}{\partial x} \right\rangle^y_\alpha + \rho_\alpha g \nabla z \qquad \Psi^y_{\alpha,y} = \frac{1}{\mu_\alpha} \left\langle \frac{\partial p_\alpha}{\partial y} \right\rangle^y_\alpha + \rho_\alpha g \nabla z$$

$$\mathbf{K}^*_r = \mathbf{K}^*_{\mathrm{tot}} {\mathbf{K}^*}^{-1}$$

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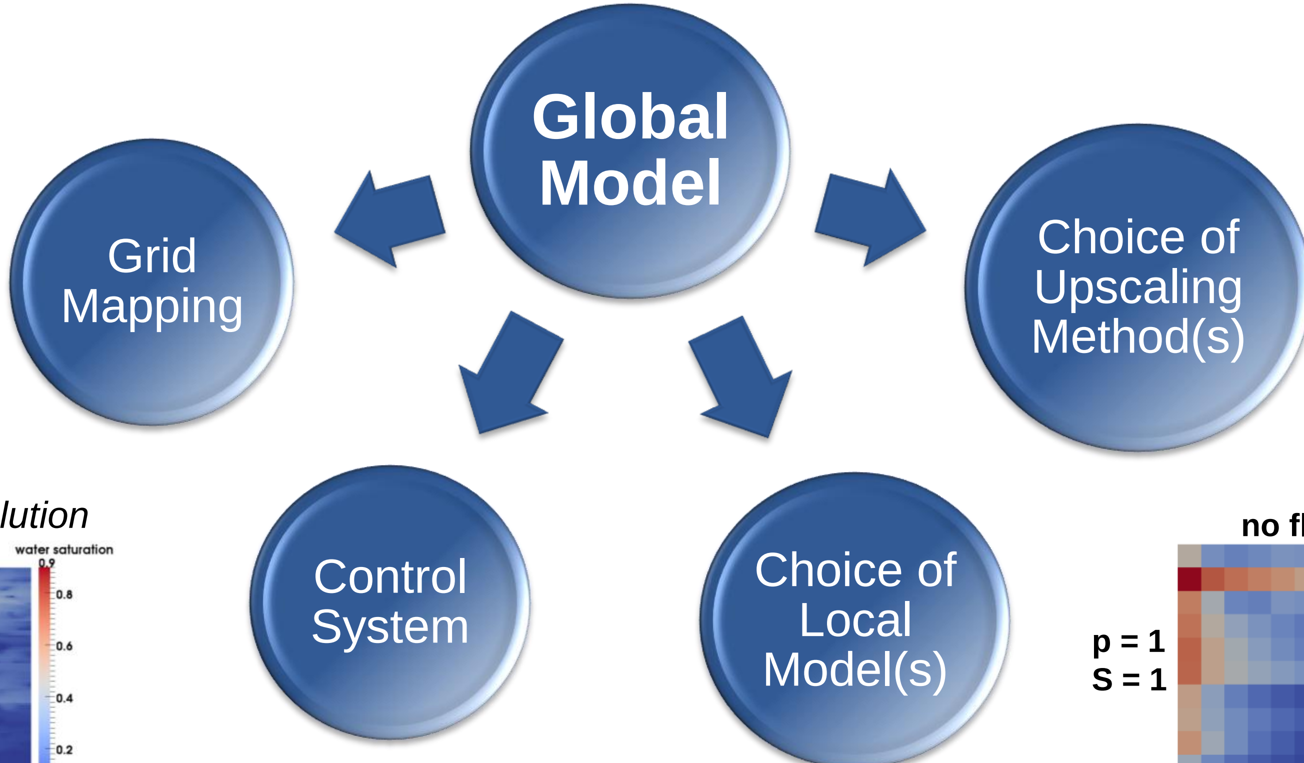
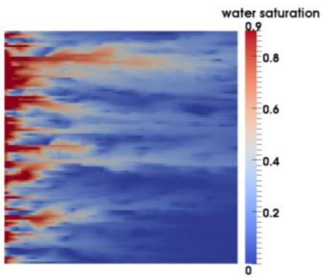
Implementation in DuMu^x

- **Aim: General framework** for use/combination of different local(-global) methods implemented in DuMu^x.

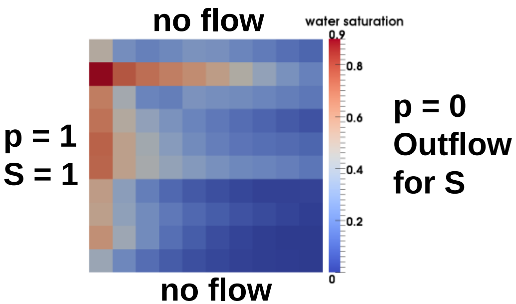
Control Structure



Reference solution

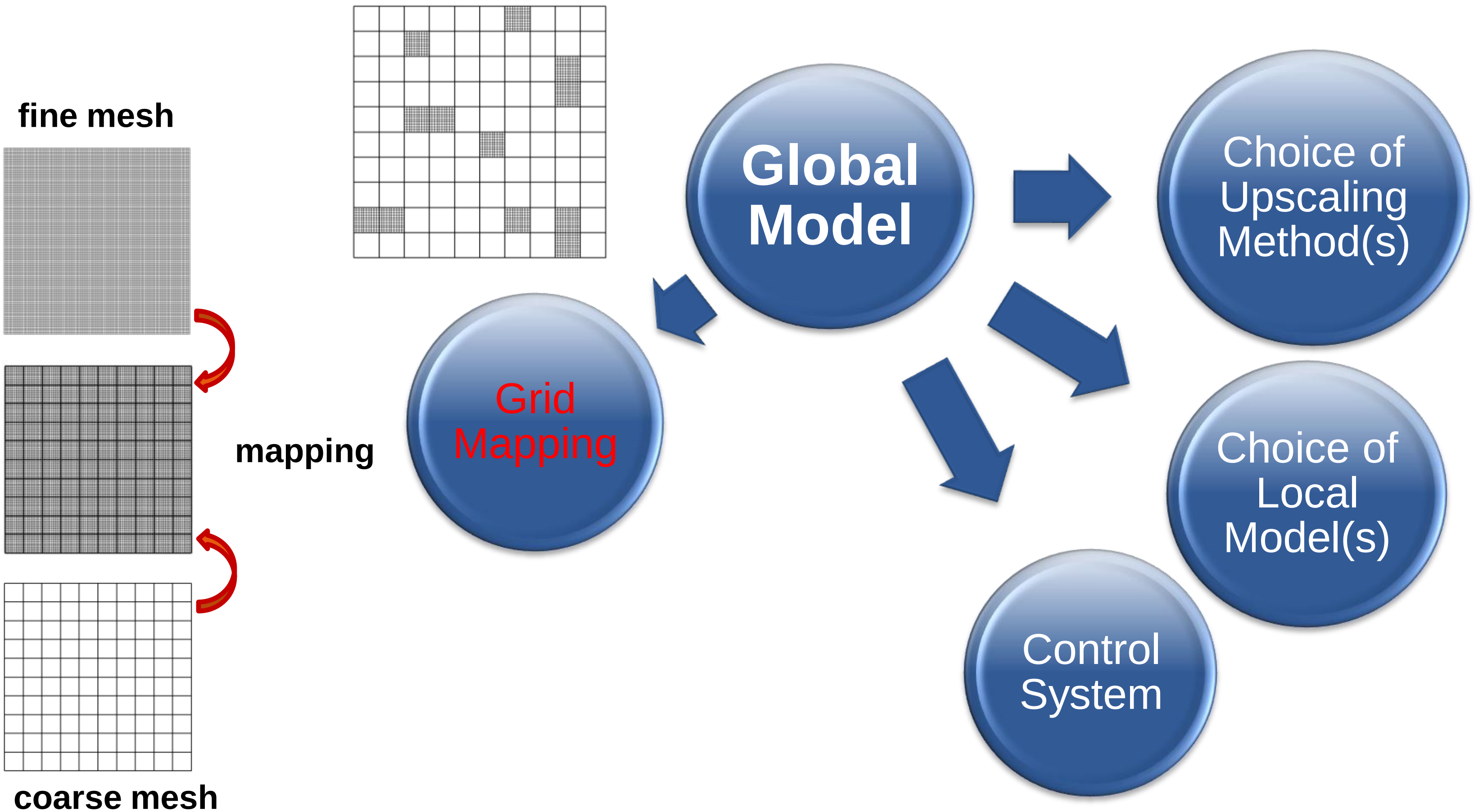


Water saturation



Water saturation

Grid Mapping

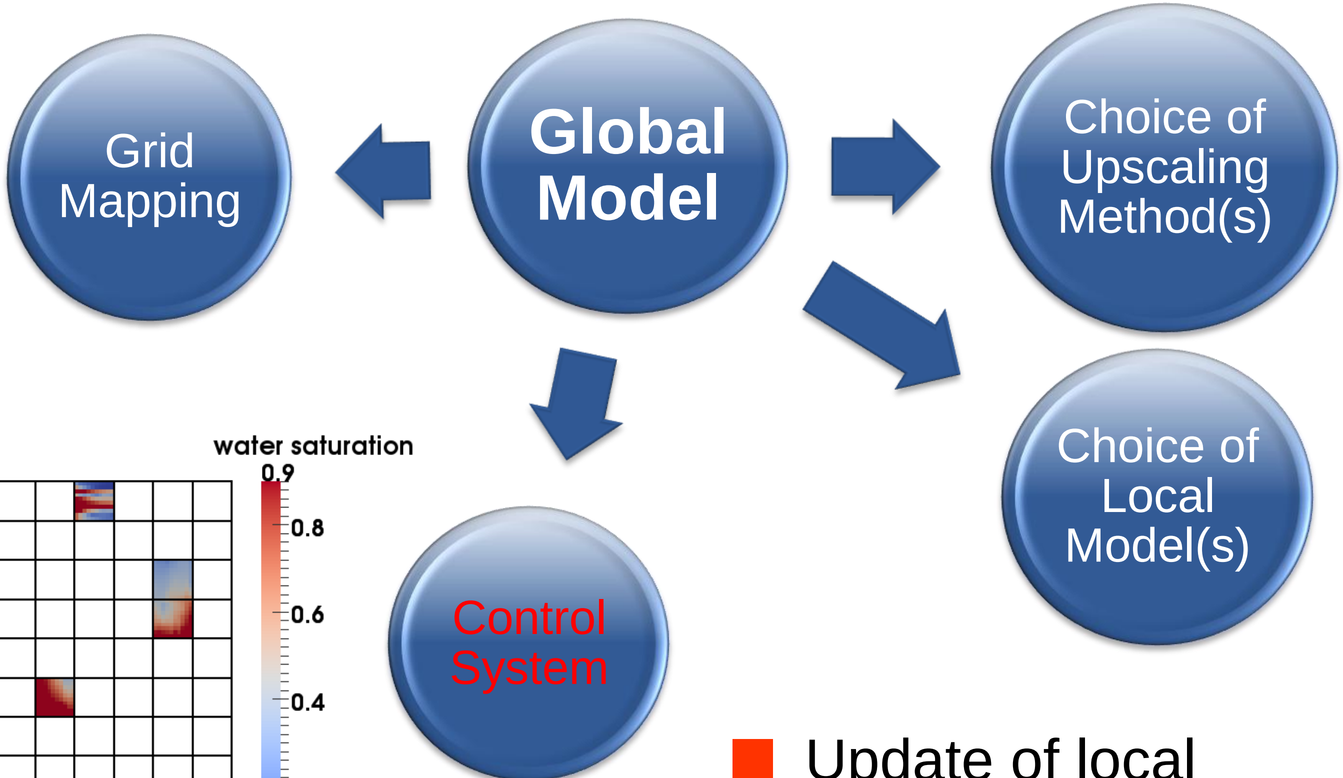


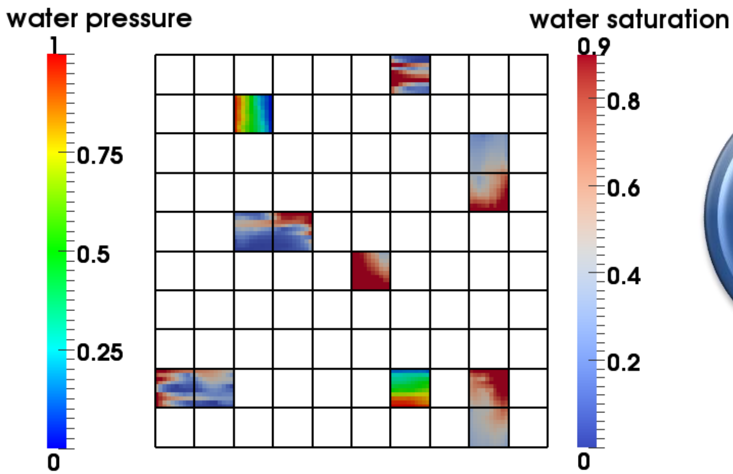
- dune-multidomaingrid

Grid Mapping Ingredient: SubdomainGenerator

```
template<class TypeTag>
void SubdomainGenerator<TypeTag>::createSubgridsOneCell(Scalar radiu
{
  gridMultiScale_.startSubDomainMarking();

  for (ElementIteratorFine eItFine = gridViewFine_.template begin<0>
  {
    int subdomainIndex = …;

    gridMultiScale_.addToSubDomain(subDomainIndex, *eItFine);
  }

  gridMultiScale_.preUpdateSubDomains();
  gridMultiScale_.updateSubDomains();
  gridMultiScale_.postUpdateSubDomains();
}
```

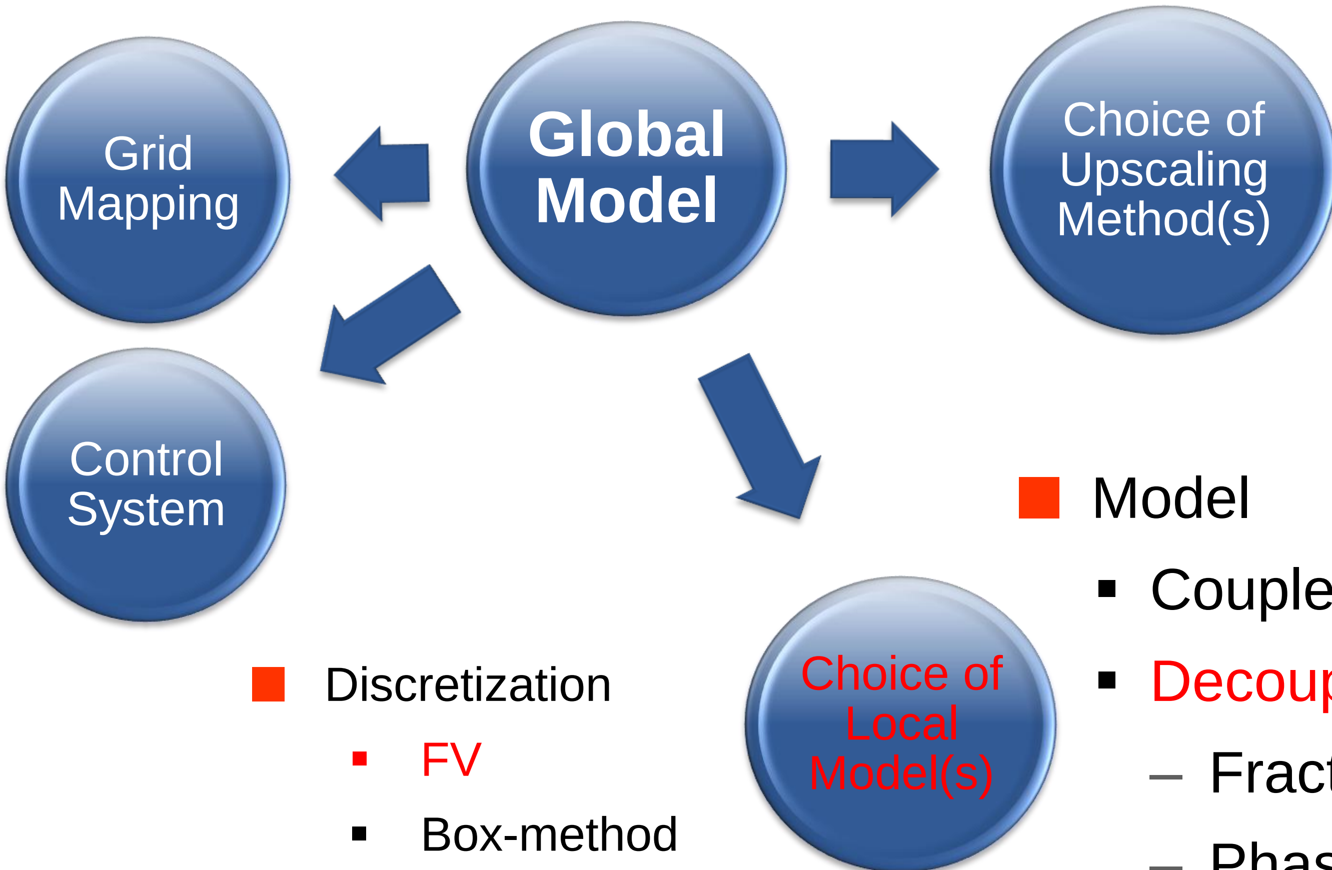
Control System





- Update of local calculations
  - Preprocessing
  - Adaptive

Choice of Local Model(s)



- Discretization
  - FV
  - Box-method
  - Mimetic FD
  - …

- Model
  - Coupled
  - Decoupled
    - Fractional flow
    - Phase pressure formulation
  - …

Ingredient: Property System

- Definition of default local problem type:

```
SET_TYPE_PROP(MultiScale,
              LocalMultiScaleBaseProblem,
              Diffusion1P<TypeTag>);
```

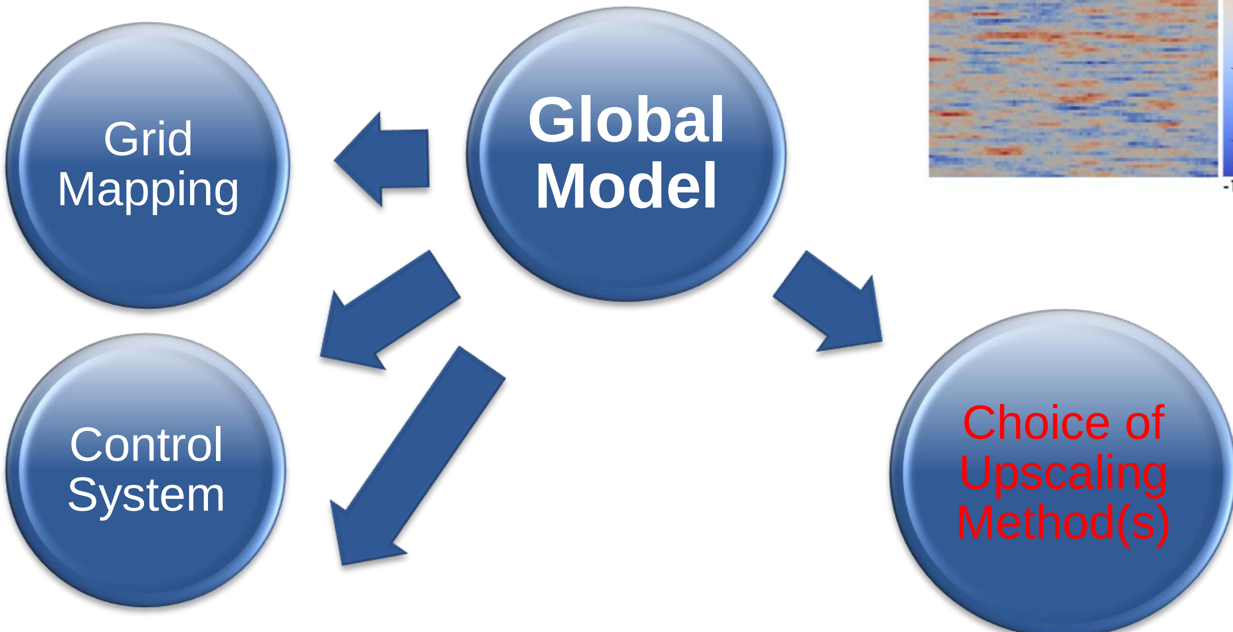
- Can be overwritten at any higher level:

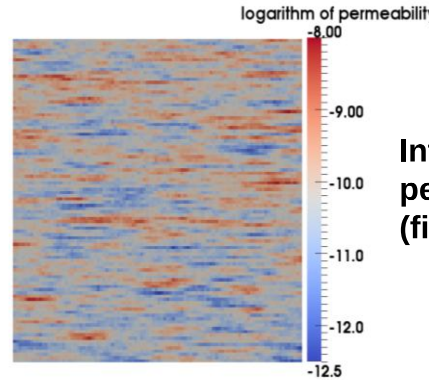
```
SET_TYPE_PROP(MyProblem,
              LocalMultiScaleBaseProblem,
              IMPET<TypeTag>);
```

- Easy typedef extraction:

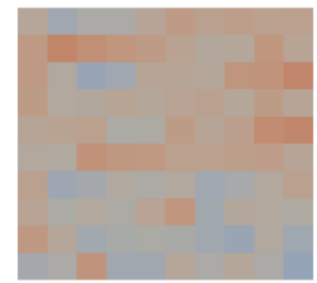
```
template<class TypeTag>
...
typedef typename GET_PROP_TYPE(TypeTag,
                   PTAG(LocalMultiScaleBaseProblem)) MyType;
```

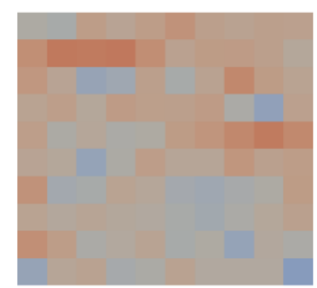
Choice of Upscaling Method(s)





Intrinsic permeability (fine scale)

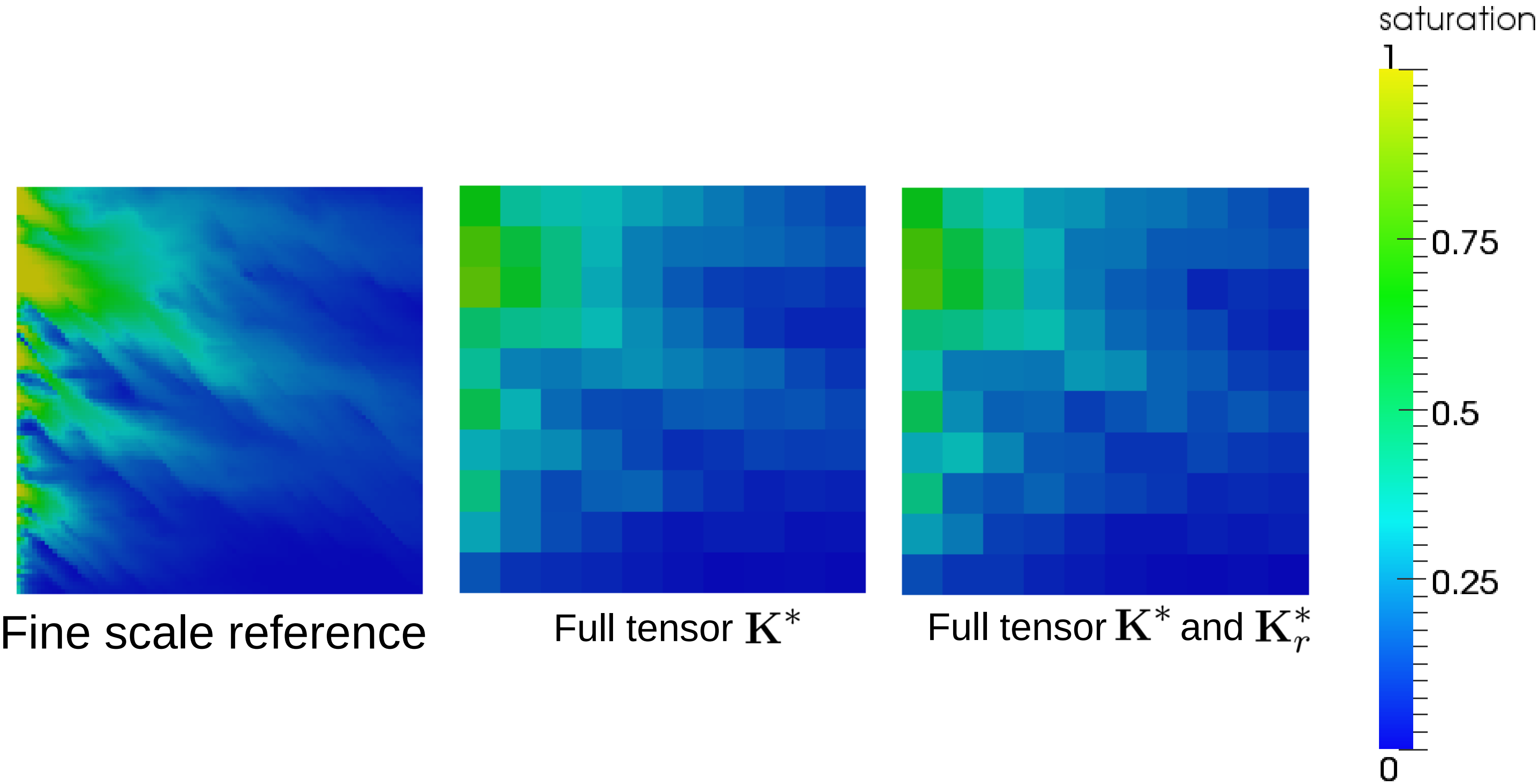




Effective permeability x- and y-direction (coarse scale)

- Methods of parameter upscaling
  - Permeabilities
  - Transmissibilities
  - Capillary pressure?

First results: 2-phase flow without p_c and gravity



Fine scale reference

Full tensor $\mathbf{K}^*$

Full tensor $\mathbf{K}^*$ and $\mathbf{K}_r^*$

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Summary

- Especially for upscaling, tensorial permeabilities are important.
- Discretizations have to be able to treat tensorial permeabilities.

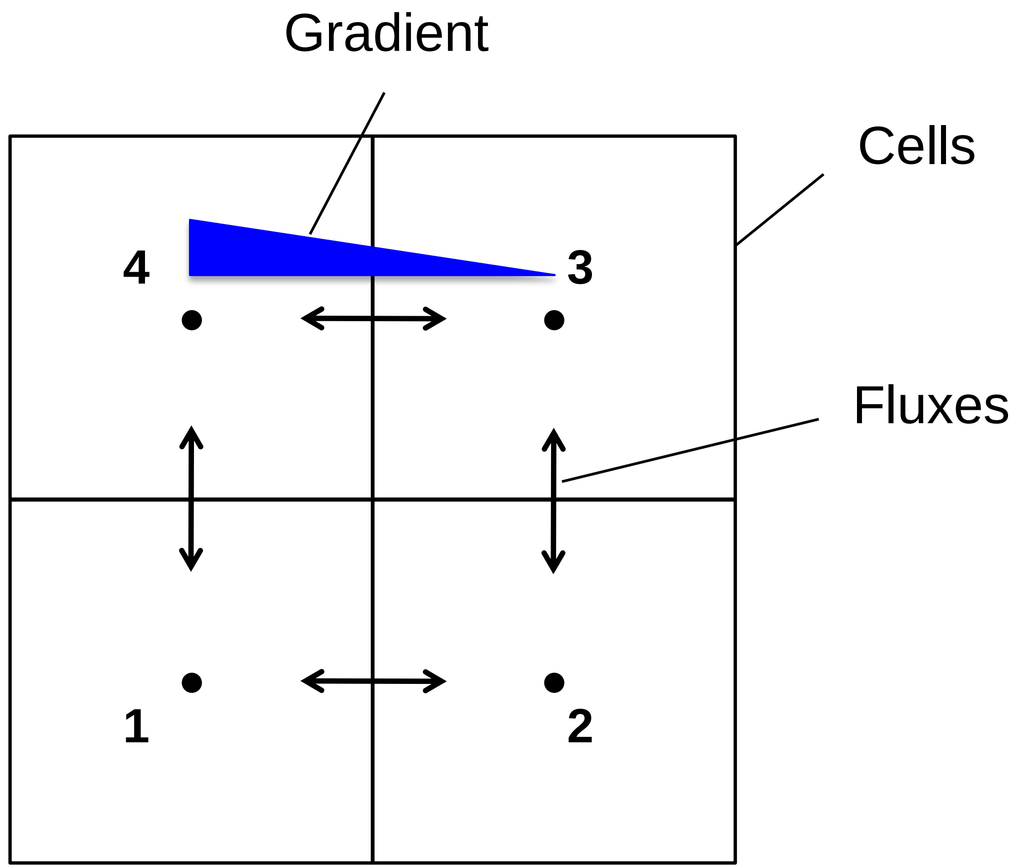
- Upscaling concept based on calculation of effective parameters.
- Intrinsic permeability upscaling is well understood.
- Relative permeability and capillary pressure upscaling not at all.

- Implementation based on dune-multidomaingrid.
- Flexible choice of local and global model and upscaling method.
- Parallelized preprocessing.

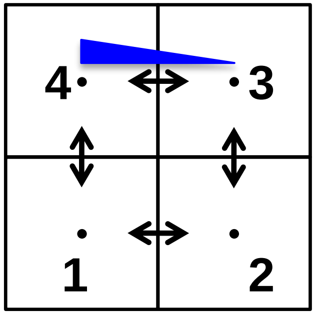
Thank you!

TPFA vs. MPFA

- TPFA:

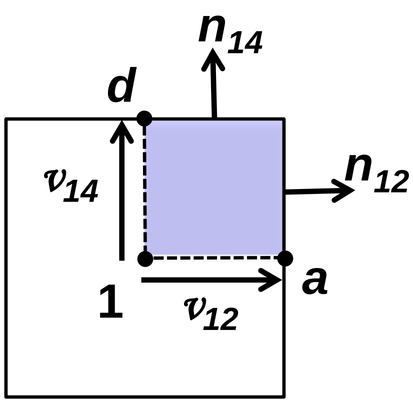


TPFA vs. MPFA



- TPFA:
  - Gradient: $$(grad\,\Phi)_{34} = \frac{\Phi_3 - \Phi_4}{\Delta x}\mathbf{n}_{34} = \begin{pmatrix} \frac{\Phi_3-\Phi_4}{\Delta x} \\ 0 \end{pmatrix}$$
  - Permeability: $$\mathbf{K}_{34} = \begin{pmatrix} \frac{2*K_{xx_3}*K_{xx_4}}{K_{xx_3}+K_{xx_4}} & 0.5*(K_{xy_3}+K_{xy_4}) \\ 0.5*(K_{yx_3}+K_{yx_4}) & \frac{2*K_{yy_3}*K_{yy_4}}{K_{yy_3}+K_{yy_4}} \end{pmatrix}$$
  - Mobility: $$\mathbf{\Lambda}_{34} = \frac{\mathbf{K}_{r34}}{\mu_{34}} = \mathbf{\Lambda}_{34}^{(\mathrm{upw})}$$
  - Flux: $$\begin{aligned} F_{34} &= (\mathbf{\Lambda}_{34}\mathbf{K}_{34}\,grad\,\Phi_{34})\,\mathbf{n}_{34}A_{34} \\ &= (\Lambda_{\mathrm{xx}_{34}}K_{\mathrm{xx}_{34}} + \Lambda_{\mathrm{xy}_{34}}K_{\mathrm{yx}_{34}})\,\Delta\Phi_{34}A_{34} \end{aligned}$$

TPFA vs. MPFA



- Potential in a cell is assumed to be linear:

$$\Phi_j(\mathbf{x}) = \nabla\Phi_j \cdot (\mathbf{x} - \mathbf{x}_{j,0}) + \Phi_{j,0}$$

  - Use this approximation and the continuity points (*a, d*) (2-d):

$$\underbrace{\begin{pmatrix} (\mathbf{x}_{1,\mathrm{a}} - \mathbf{x}_{1,0})^T \\ (\mathbf{x}_{1,\mathrm{d}} - \mathbf{x}_{1,0})^T \end{pmatrix}}_{\mathbf{X}} \nabla\Phi_1 = \begin{pmatrix} \Phi_{1,\mathrm{a}} - \Phi_{1,0} \\ \Phi_{1,\mathrm{d}} - \Phi_{1,0} \end{pmatrix}$$

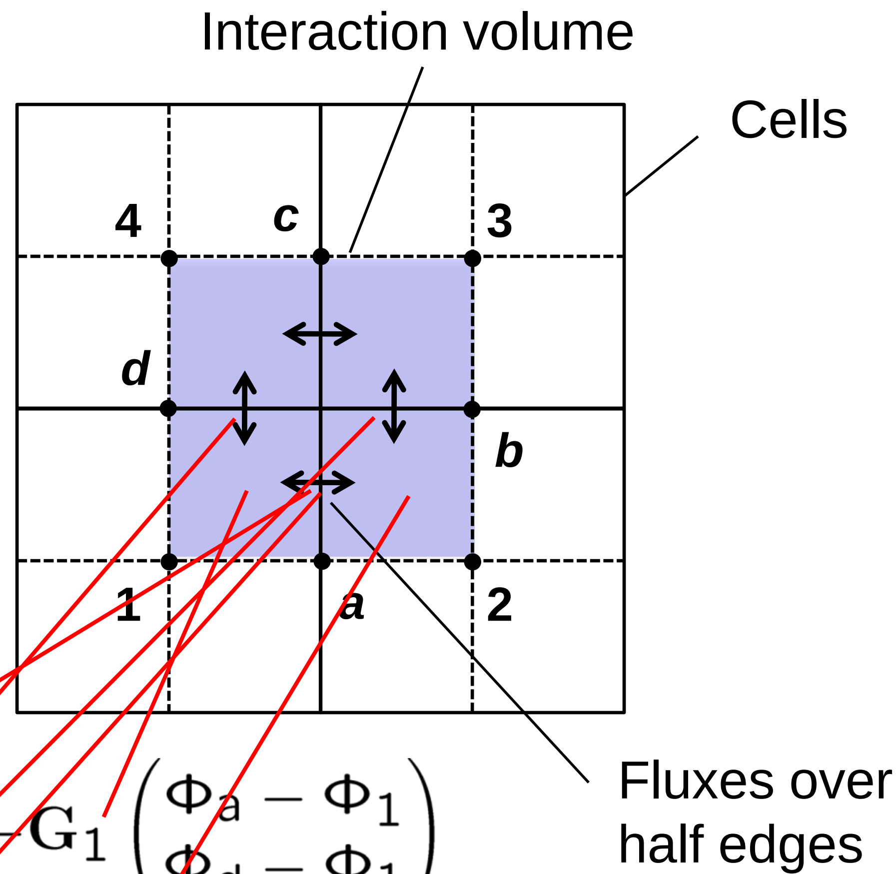
  - After some reformulation the gradient can be approximated as:

$$\nabla\Phi_1 = \frac{1}{V_\mathrm{t}}\Big(\nu_{12}(\Phi_{1,\mathrm{a}} - \Phi_{1,0}) + \nu_{14}(\Phi_{1,\mathrm{d}} - \Phi_{1,0})\Big)$$

$$\mathbf{X}^{-1} = \frac{1}{V_\mathrm{t}}[\nu_{12}, \nu_{14}]$$

TPFA vs. MPFA

- MPFA-O:
  - Introduce pressures/potentials at face centers (*a*, *b*, *c*, *d*)
  - Formulate fluxes over half cell edges for every quarter of the interaction volume:

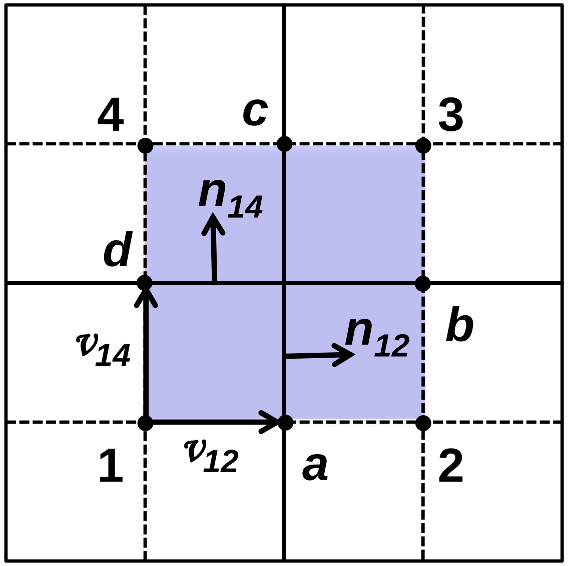


$$\begin{pmatrix} f_{12} \\ f_{14} \end{pmatrix} = -\mathbf{G}_1 \begin{pmatrix} \Phi_a - \Phi_1 \\ \Phi_d - \Phi_1 \end{pmatrix}$$

$$\begin{pmatrix} f_{23} \\ f_{21} \end{pmatrix} = -\mathbf{G}_2 \begin{pmatrix} \Phi_b - \Phi_2 \\ \Phi_2 - \Phi_a \end{pmatrix}$$

$$\ldots$$

TPFA vs. MPFA



- MPFA-O:
  - $\mathbf{G}_1$ includes $\mathbf{K}$, $\boldsymbol{\Lambda}$ and geometric information

$$\nu_{12} = \frac{(\mathbf{x}_a - \mathbf{x}_1)}{|(\mathbf{x}_a - \mathbf{x}_1)|} A_{14}$$

$$\mathbf{G}_1 = \frac{1}{V_t} \begin{pmatrix} A_{12}\mathbf{n}_{12}^T \boldsymbol{\Lambda}_1 \mathbf{K}_1 \nu_{12} & A_{12}\mathbf{n}_{12}^T \boldsymbol{\Lambda}_1 \mathbf{K}_1 \nu_{14} \\ A_{14}\mathbf{n}_{14}^T \boldsymbol{\Lambda}_2 \mathbf{K}_2 \nu_{12} & A_{14}\mathbf{n}_{14}^T \boldsymbol{\Lambda}_2 \mathbf{K}_2 \nu_{14} \end{pmatrix}$$

  - Structured grid

$$\mathbf{G}_1 = \frac{1}{V_t} \begin{pmatrix} A_{12}\mathbf{n}_{12}^T \boldsymbol{\Lambda}_1 \mathbf{K}_1 A_{14}\mathbf{n}_{12} & A_{12}\mathbf{n}_{12}^T \boldsymbol{\Lambda}_1 \mathbf{K}_1 A_{12}\mathbf{n}_{14} \\ A_{14}\mathbf{n}_{14}^T \boldsymbol{\Lambda}_2 \mathbf{K}_2 A_{14}\mathbf{n}_{12} & A_{14}\mathbf{n}_{14}^T \boldsymbol{\Lambda}_2 \mathbf{K}_2 A_{12}\mathbf{n}_{14} \end{pmatrix}$$

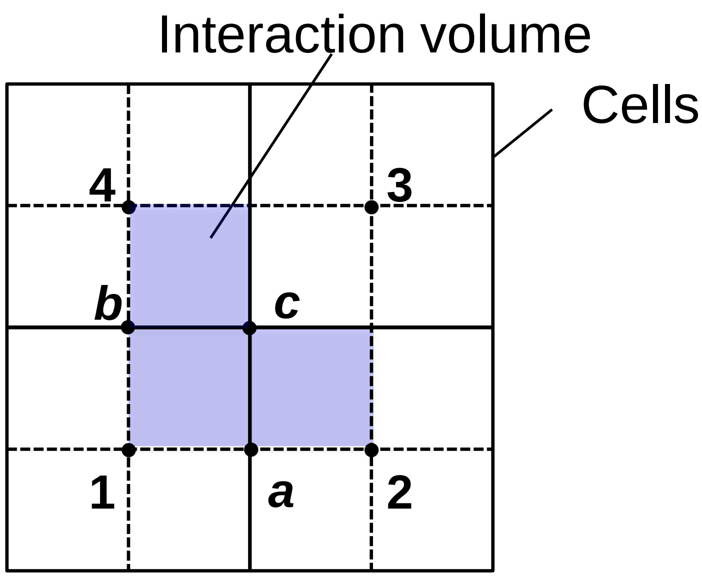
TPFA vs. MPFA

- MPFA-O:
  - Fluxes: $$f_{12} = -\frac{1}{V_t} A_{12} A_{14} (\Lambda_{xx} K_{xx} + \Lambda_{xy} K_{yx})(\Phi_a - \Phi_1) - \frac{1}{V_t} A_{12} A_{12} (\Lambda_{xx} K_{xy} + \Lambda_{xy} K_{yy})(\Phi_d - \Phi_1)$$
  - Solve the system of equations arising from a flux balance to get the Transmissibility matrix **T**:

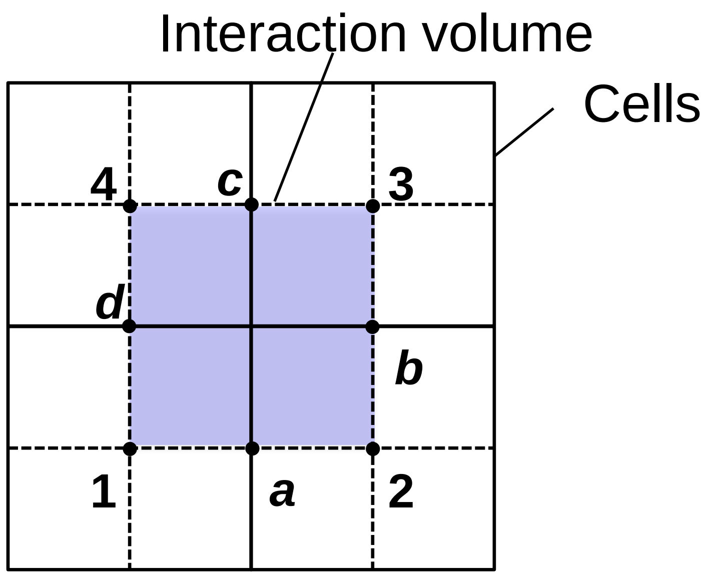
$$\begin{pmatrix} f_{12} \\ f_{23} \\ f_{34} \\ f_{41} \end{pmatrix} = \mathbf{T} \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \Phi_4 \end{pmatrix}$$

$$f_{12} = f_{21}$$
$$f_{23} = f_{32}$$
$$f_{34} = f_{43}$$
$$f_{41} = f_{14}$$

TPFA vs. MPFA



L-Method



O-Method

TPFA vs. MPFA

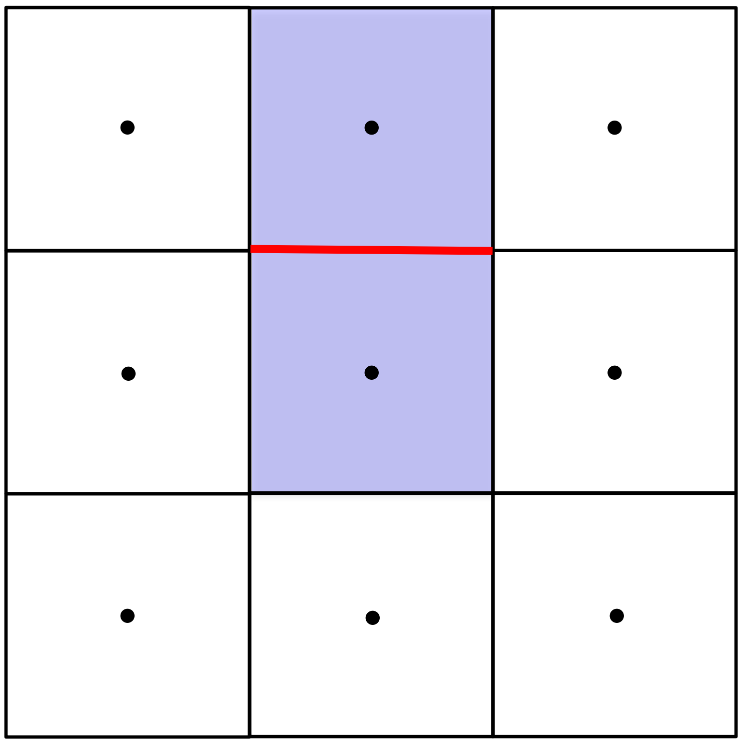
- TPFA
  - only for structured grids (CCFV)
- MPFA-O
  - Accurate for unstructured grids (CCFV)

TPFA vs. MPFA

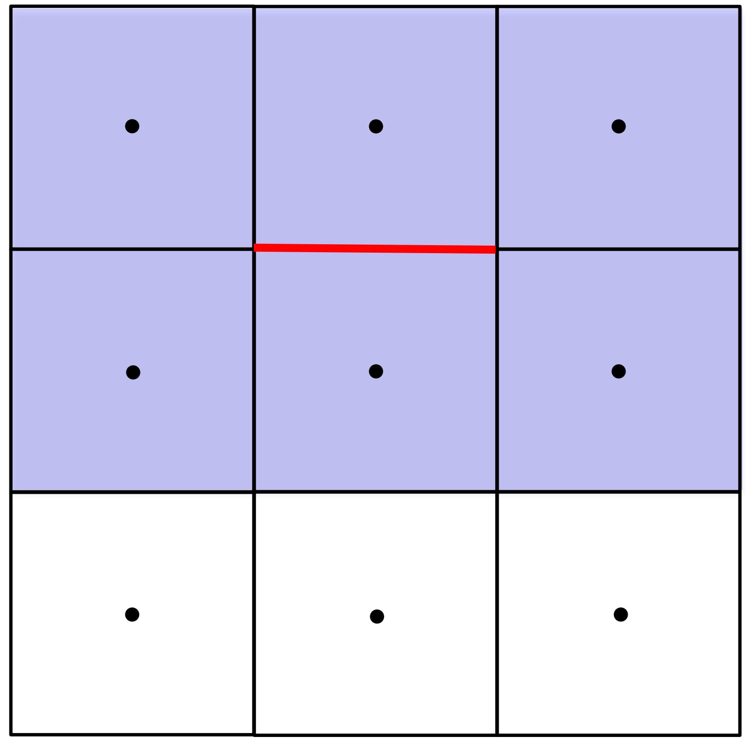
- TPFA
  - only for structured grids (CCFV)
  - Face flux with information of the 2 neighbor cells
- MPFA-O
  - Accurate for unstructured grids (CCFV)
  - Face flux with information of 6 (2-D) surrounding cells

TPFA vs. MPFA

- TPFA



- MPFA-O



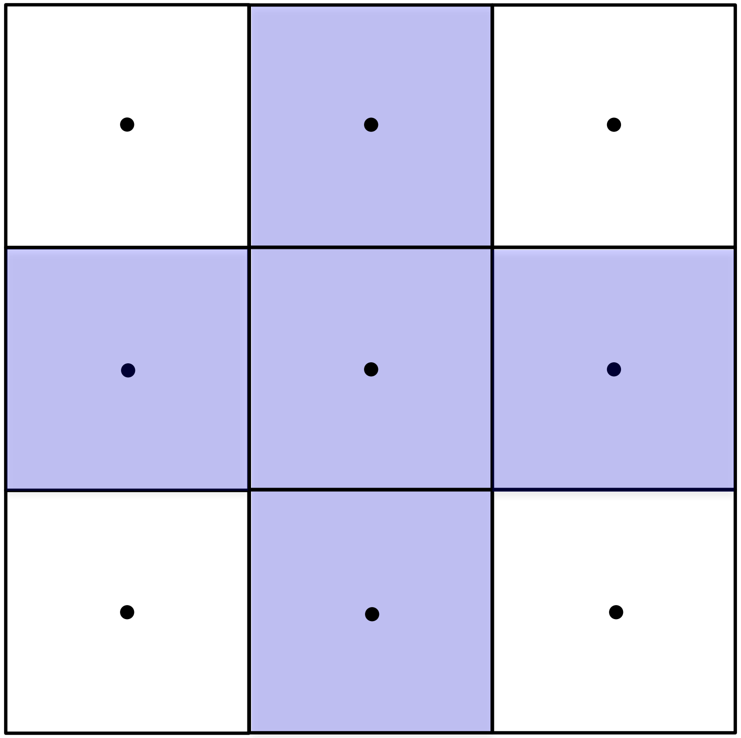
TPFA vs. MPFA

- TPFA
  - only for structured grids (CCFV)
  - Face flux with information of the 2 neighbor cells
  - 5-point stencil → problem if flux is not aligned with the principal grid axes

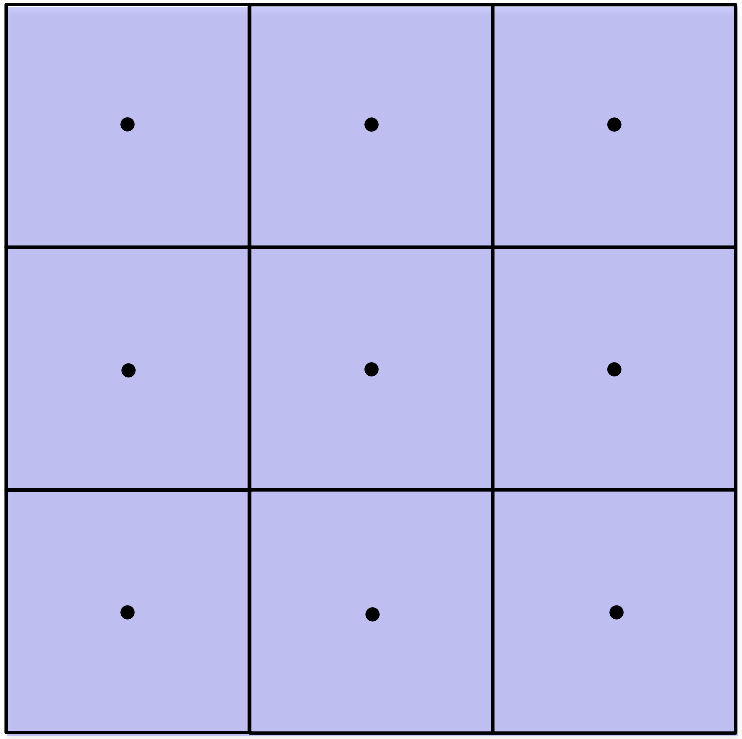
- MPFA-O
  - Accurate for unstructured grids (CCFV)
  - Face flux with information of 6 (2-D) surrounding cells
  - 9-point stencil → diagonal effects accounted for

TPFA vs. MPFA

- TPFA



- MPFA-O



TPFA vs. MPFA

- TPFA
  - only for structured grids (CCFV)
  - Face flux with information of the 2 neighbor cells
  - 5-point stencil → problem if flux is not aligned with the principal grid axes
  - Not able to account properly for the full tensor effects

- MPFA-O
  - Accurate for unstructured grids (CCFV)
  - Face flux with information of 6 (2-D) surrounding cells
  - 9-point stencil → diagonal effects accounted for
  - Accounts for full tensor effects

Small Summary

- It can be necessary to use full tensor coefficients
- A MPFA-O method is able to treat them

➡ We need a MPFA method for two-phase flow with capillary pressure and gravity!

Treatment of tensor coefficients with MPFA-O

- Two-phase flow formulation (*Hotetit & Firoozabadi)*:
  - Definition of potentials:

$$\Phi_w = p_\mathrm{w} + \rho_\mathrm{w} g z \qquad \Phi_n = p_\mathrm{n} + \rho_\mathrm{n} g z$$

$$\Phi_c = \Phi_\mathrm{n} - \Phi_\mathrm{w} = p_\mathrm{c} + (\rho_\mathrm{n} - \rho_\mathrm{w}) g z$$

  - Total velocity:

$$\mathbf{v}_\mathrm{t} = \mathbf{v}_\mathrm{w} + \mathbf{v}_\mathrm{n} \qquad \boldsymbol{\Lambda}_\mathrm{t} = \boldsymbol{\Lambda}_\mathrm{w} - \boldsymbol{\Lambda}_\mathrm{n}$$

$$\mathbf{v}_\mathrm{t} = \underbrace{-\boldsymbol{\Lambda}_\mathrm{t} \mathbf{K} \nabla \Phi_\mathrm{w}}_{\mathbf{v}_{(\mathrm{adv})}} \underbrace{+\boldsymbol{\Lambda}_\mathrm{t}^{-1} \boldsymbol{\Lambda}_\mathrm{n} \boldsymbol{\Lambda}_\mathrm{t} \nabla \Phi_\mathrm{c}}_{\mathbf{v}_{(\mathrm{cap})}} \qquad \boldsymbol{\Lambda}_\alpha = \frac{1}{\mu_\alpha} \mathbf{K}_{\mathrm{r}_\alpha}$$

  - Pressure equation:

$$\nabla \cdot \mathbf{v}_\mathrm{t} = q_\mathrm{t}$$

Treatment of tensor coefficients with MPFA-O

- Discretization of the pressure equation with finite volumes and MPFA:

$$\int_V \nabla \cdot \mathbf{v}_\mathrm{t} \, dV = \sum_i \left(F_{(\mathrm{adv})_i} + F_{(\mathrm{cap})_i} \right) = q_\mathrm{t} V$$

  - Total transmissibility $\mathbf{T}_\mathrm{t}$ as operator for $A\,\mathbf{n}\boldsymbol{\Lambda}_\mathrm{t}\mathbf{K}\nabla$:

$$\underbrace{\mathbf{F}_{(\mathrm{adv})_\mathrm{w}} = \mathbf{T}_\mathrm{t}\boldsymbol{\Phi}_\mathrm{w}}_{\text{advective flux}} \qquad \underbrace{\mathbf{F}_{(\mathrm{cap})} = \boldsymbol{\Lambda}_\mathrm{t}^{-1}\boldsymbol{\Lambda}_\mathrm{n}\mathbf{T}_\mathrm{t}\boldsymbol{\Phi}_\mathrm{c}}_{\text{capillary flux}}$$

Treatment of tensor coefficients with MPFA-O

- Calculation of phase velocities:

$$\mathbf{v}_{\mathrm{w}_i} = \boldsymbol{\Lambda}_{\mathrm{t}_i}^{(\mathrm{upw})^{-1}} \boldsymbol{\Lambda}_{\mathrm{w}_i}^{(\mathrm{upw})} \left(\mathbf{T}_{\mathrm{t}} \boldsymbol{\Phi}_{\mathrm{w}}\right)_i \frac{\mathbf{n}_i}{A_i}$$

$$\mathbf{v}_{\mathrm{n}_i} = \left[\boldsymbol{\Lambda}_{\mathrm{t}_i}^{(\mathrm{upw})^{-1}} \boldsymbol{\Lambda}_{\mathrm{n}_i}^{(\mathrm{upw})} \left(\mathbf{T}_{\mathrm{t}} \boldsymbol{\Phi}_{\mathrm{w}}\right)_i + \boldsymbol{\Lambda}_{\mathrm{t}_i}^{-1} \boldsymbol{\Lambda}_{\mathrm{n}_i} \left(\mathbf{T}_{\mathrm{t}} \boldsymbol{\Phi}_{(\mathrm{cap})}\right)_i\right] \frac{\mathbf{n}_i}{A_i}$$

- Discretisation of the saturation equation with finite volumes:

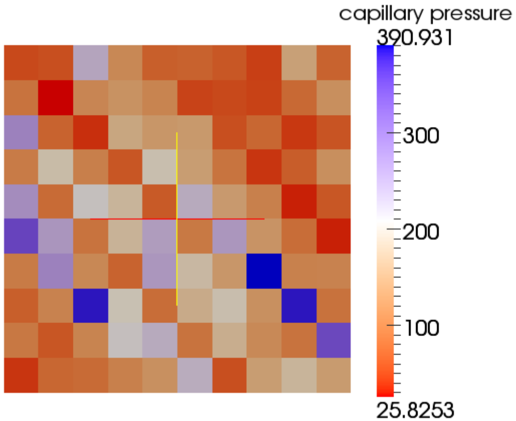
$$\varphi \frac{\partial S_{\mathrm{w}}}{\partial t} V + \sum_i \left(\mathbf{v}_{\mathrm{w}} \cdot n\right)_i A_i = q_i V$$

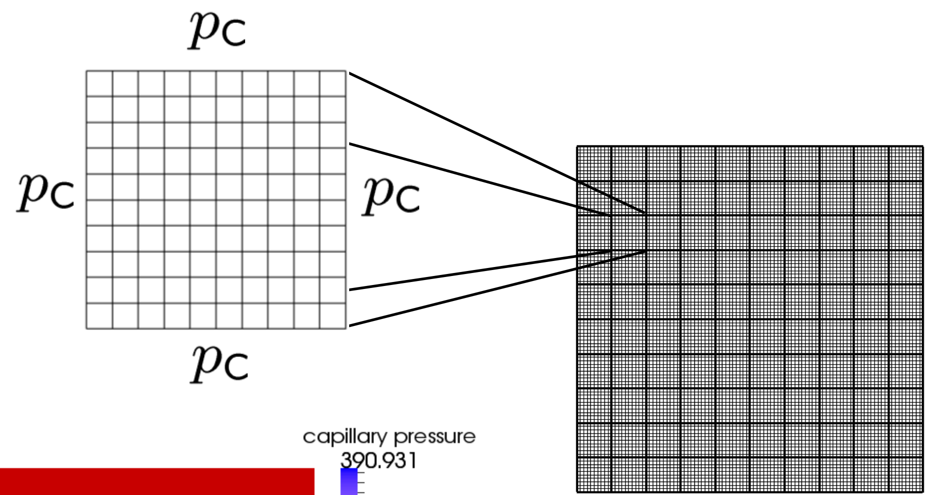
How can tensor parameters be determined?

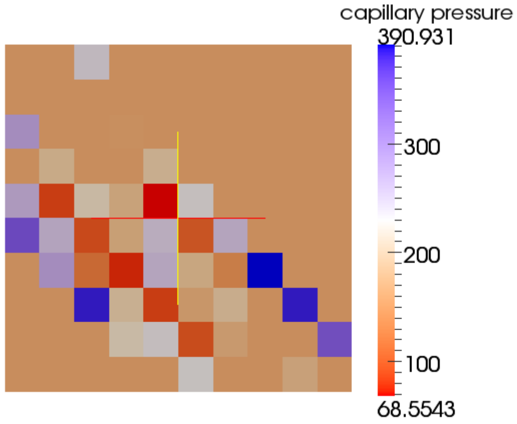
- Can arise from upscaling
  - Analytical methods (only simple structures)
  - Numerical methods
  - …
- Well tested methods for intrinsic permeability upscaling available
- Tensor relative permeabilities?

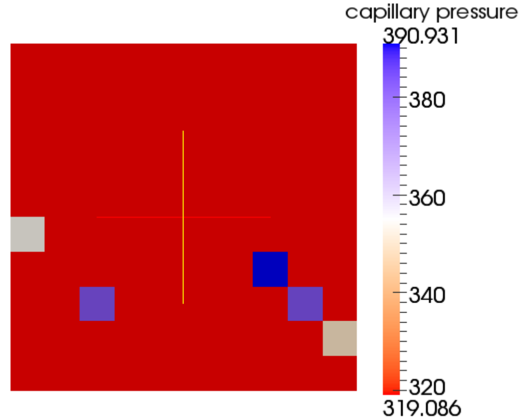
Relative permeability upscaling

- Determine initial saturation distribution from capillary equilibrium









Example of an upscaled relative permeability function